TERRESTRIAL SPHERE

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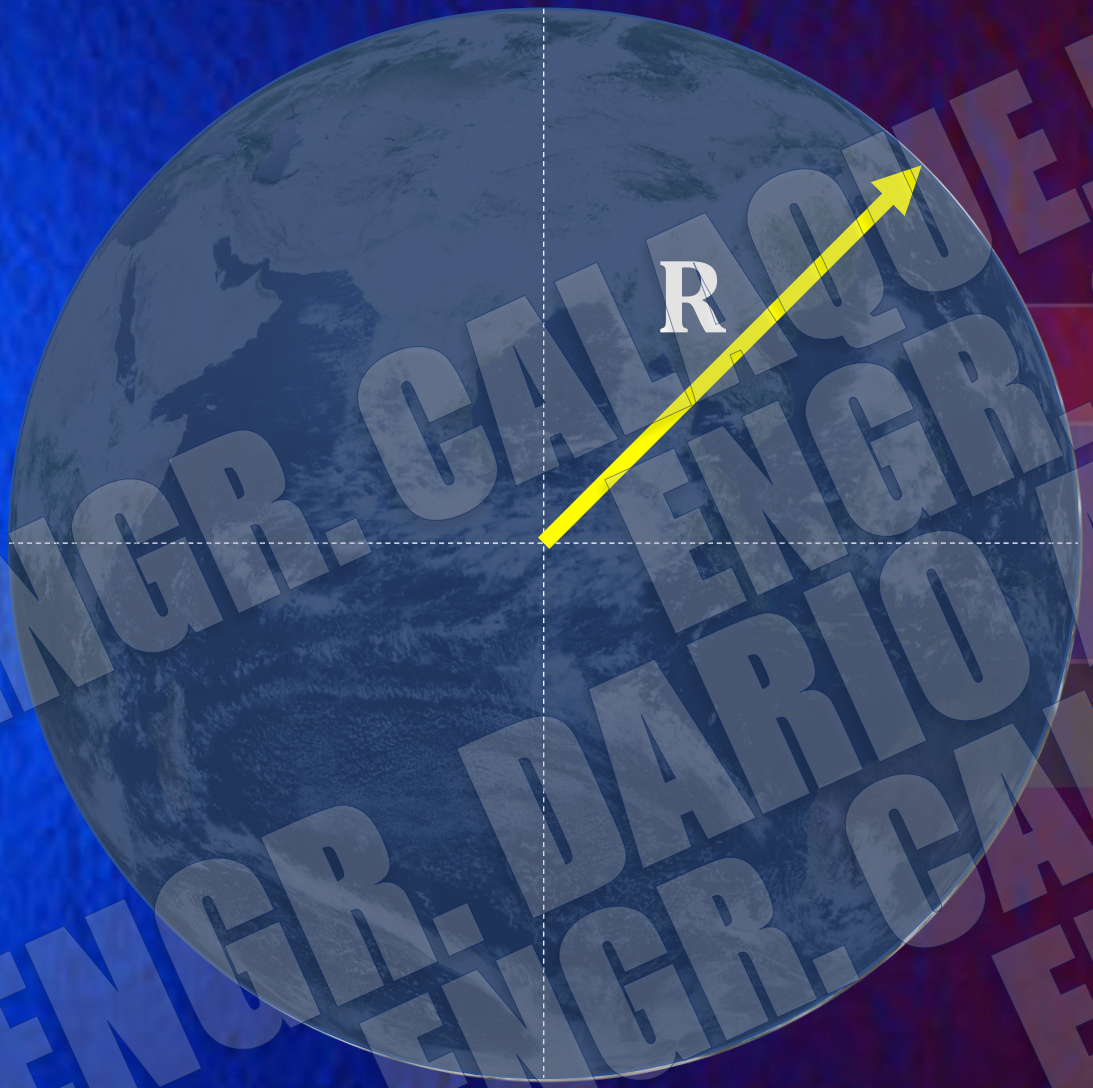
TERRESTRIAL SPHERE



RADIUS OF EARTH
= 3959MILES
= 6371KM

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TERRESTRIAL SPHERE

RADIUS OF EARTH = 3959MILES

$$\frac{7.2^0}{360^0} = \frac{800km}{C}$$

$$\frac{1}{50} = \frac{800km}{C}$$

$$C = 40,000km$$

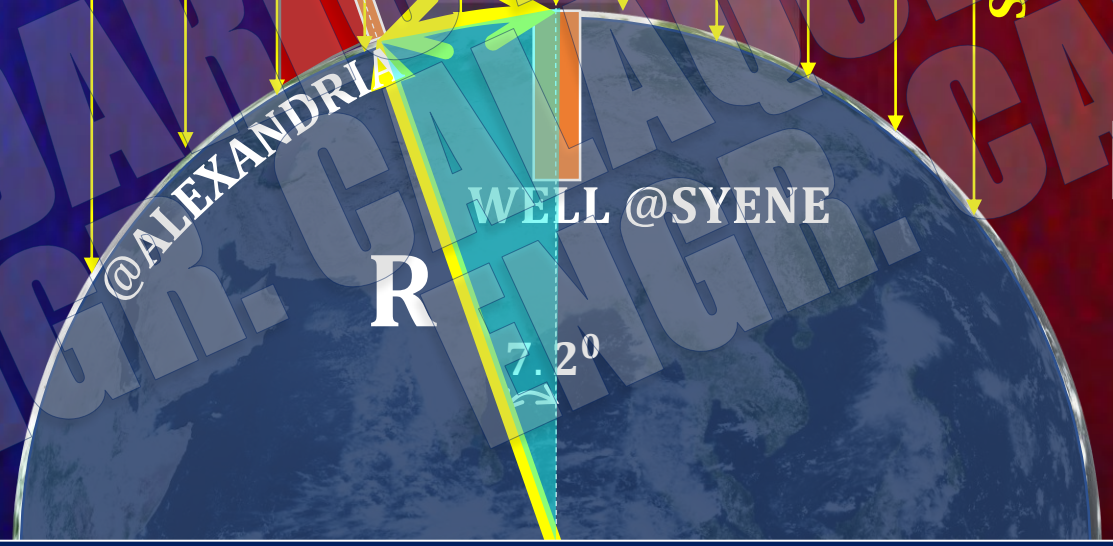
$$2\pi R = 40,000$$

$$R = 6366km$$

0.08%error

$6366km + 5km = 6371KM$?

240B.C
ERATOSTHENES
Father of Geography



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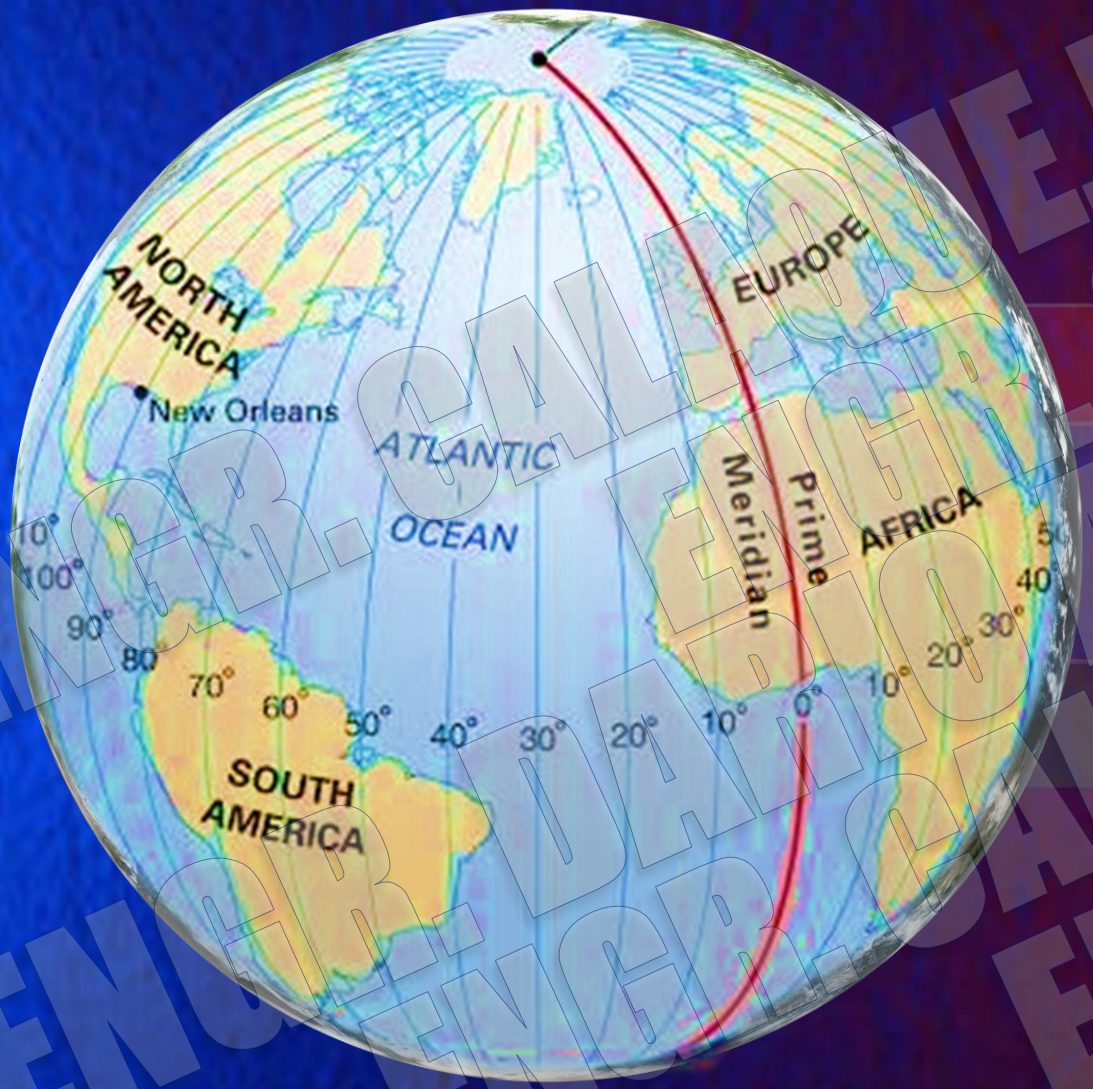

NORTH AND SOUTH POLES

= ARE THE POLES OF THE EARTH WHICH ARE ON ITS AXIS

MERIDIANS OR LONGITUDES

= ARE CURVES PERPENDICULAR TO THE EQUATOR. LONGITUDE OF A POINT IS THE ANGLE BETWEEN THE PLANE OF ITS MERIDIAN AND THE PLANE OF THE GREENWICH MERIDIAN. IT IS MEASURED FROM $0^0 TO\ 180^0$ EAST OR WEST. MERIDIANS ARE ALL GREAT CIRCLES WHILE PARALLELS ARE NOT UNLESS IT IS AT THE EQUATOR

PRIME MERIDIAN

(ALSO KNOWN AS GREENWICH MERIDIAN)

– THE MERIDIAN AGREED INTERNATIONALLY IN 1864 TO BE OF LONGITUDE 0^0. IT PASSES THROUGH FORMER LONDON OBSERVATORY AT GREENWICH.

BEARINGS OR COURSES

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ENGR. DARIO

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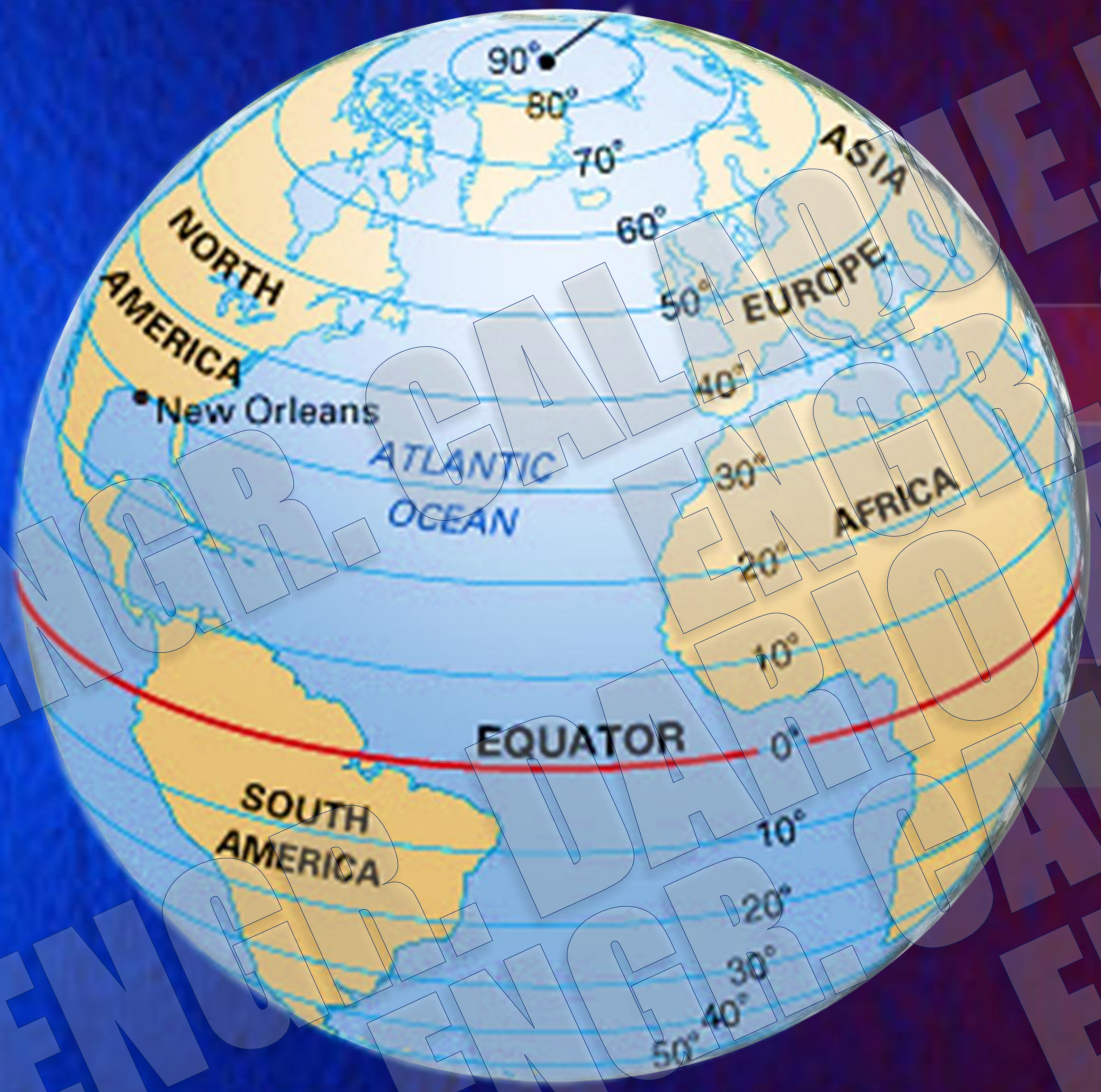

EQUATOR

= AN IMAGINARY LINE PASSING THROUGH THE CENTER OF THE EARTH AND PERPENDICULAR TO THE AXIS OF THE EARTH

PARALLELS OR LATITUDES

= ARE CURVES PARALLEL TO THE EQUATOR. LATITUDE OF A POINT IS THE SMALLEST ANGLE FORMED BETWEEN THE RADIUS FROM THE POINT TO THE EARTH'S CENTER AND THE PLANE OF THE EQUATOR. IT IS MEASURED FROM 0^0 *TO* 90^0 *NORTH OR SOUTH*

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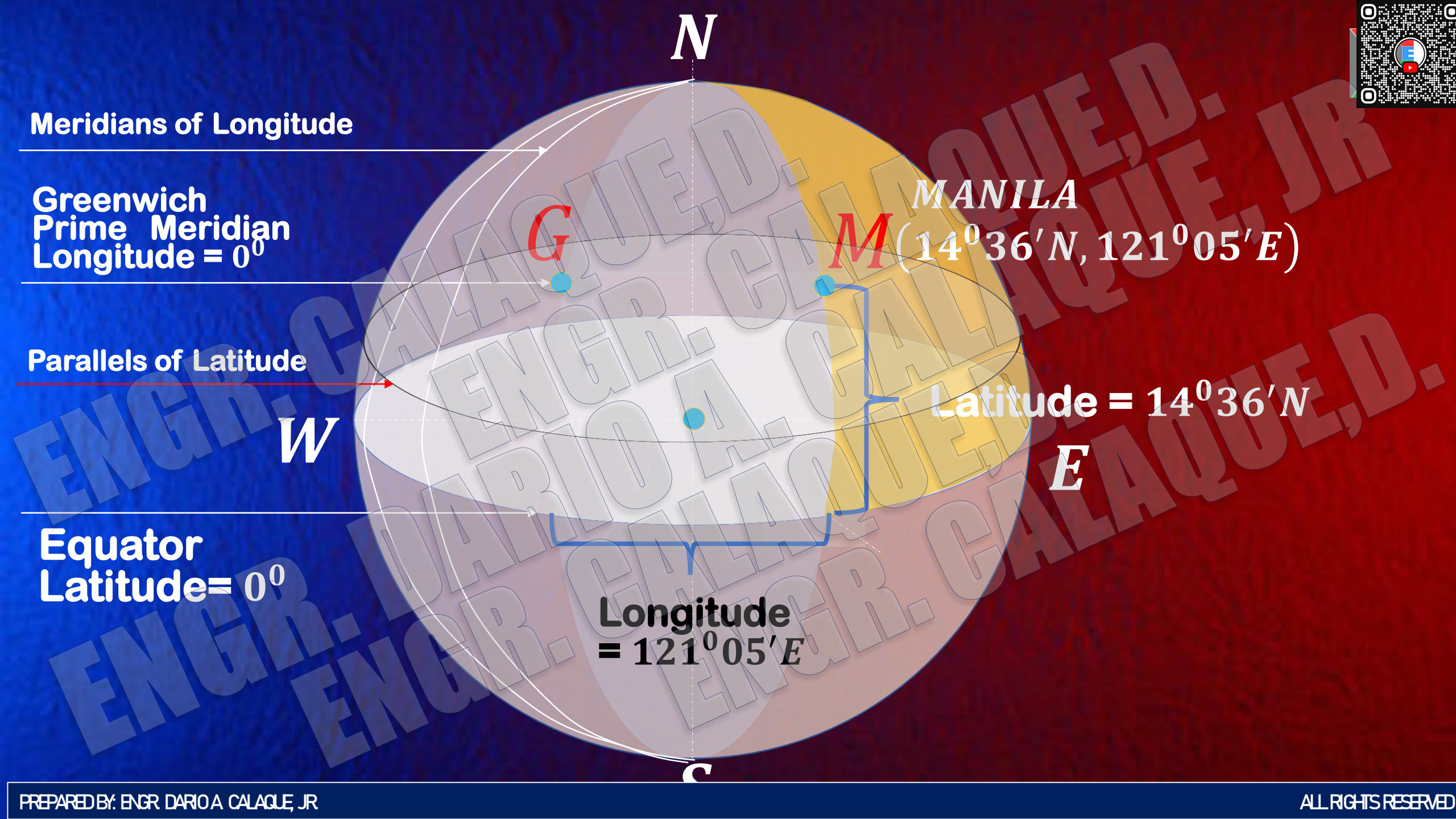

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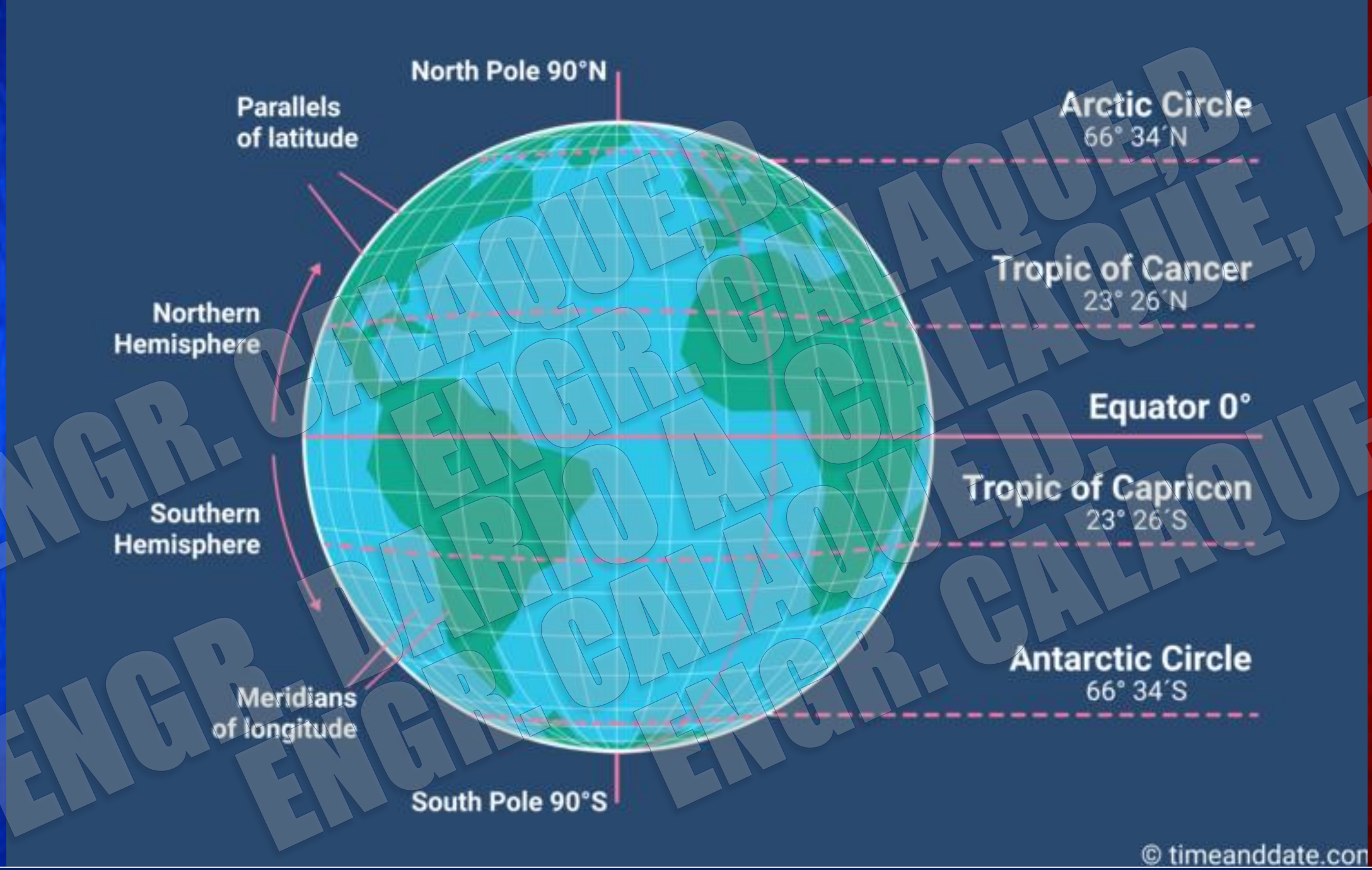

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PAST CE BOARD EXAM:

IF THE LONGITUDE OF TOKYO IS 139^0E AND THAT OF MANILA IS 121^0E, WHAT IS THE TIME DIFFERENCE BETWEEN TOKYO AND MANILA?

A) 1HOUR AND 12MINUTES

B) 1HOUR AND 5MINUTES

C) 1HOUR AND 8MINUTES

D) 1HOUR AND 10MINUTES

ALTERNATE SOLUTION:

$$\frac{Difference\ in\ Time}{24} = \frac{Difference\ in\ Longitude}{360}$$

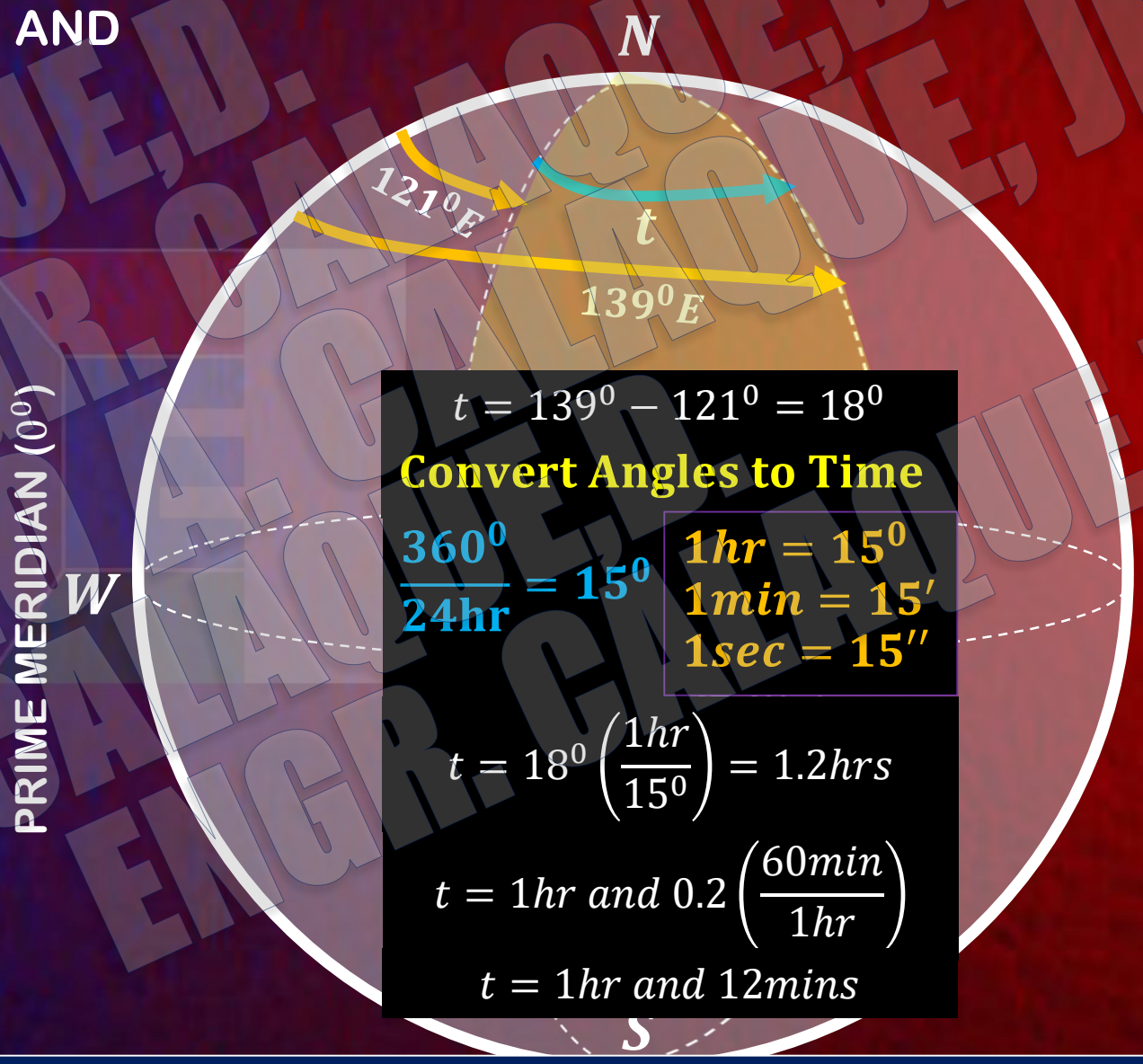
$$\frac{Difference\ in\ Time}{24} = \frac{139^0 - 121^0}{360}$$

$$Difference\ in\ Time = 1.2hrs$$

$$Difference\ in\ Time = 1hr\ and\ 0.2\left(\frac{60min}{1hr}\right)$$

$$Difference\ in\ Time = 1hr\ and\ 12mins$$

SOLUTION:



$$t = 139^0 - 121^0 = 18^0$$

Convert Angles to Time

$$\frac{360^0}{24hr} = 15^0$$

$$1hr = 15^0$$
$$1min = 15'$$
$$1sec = 15''$$

$$t = 18^0\left(\frac{1hr}{15^0}\right) = 1.2hrs$$

$$t = 1hr\ and\ 0.2\left(\frac{60min}{1hr}\right)$$

$$t = 1hr\ and\ 12mins$$

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PAST CE BOARD EXAM:

IF GREENWICH MEAN TIME (GMT) IS 6A.M., WHAT IS THE TIME AT A PLACE LOCATED 30^0 EAST LONGITUDE?

A) 7A.M

B) 8A.M

C) 9A.M

D) 4A.M

ALTERNATE SOLUTION:

$$\frac{Difference\ in\ Time}{24} = \frac{Difference\ in\ Longitude}{360}$$

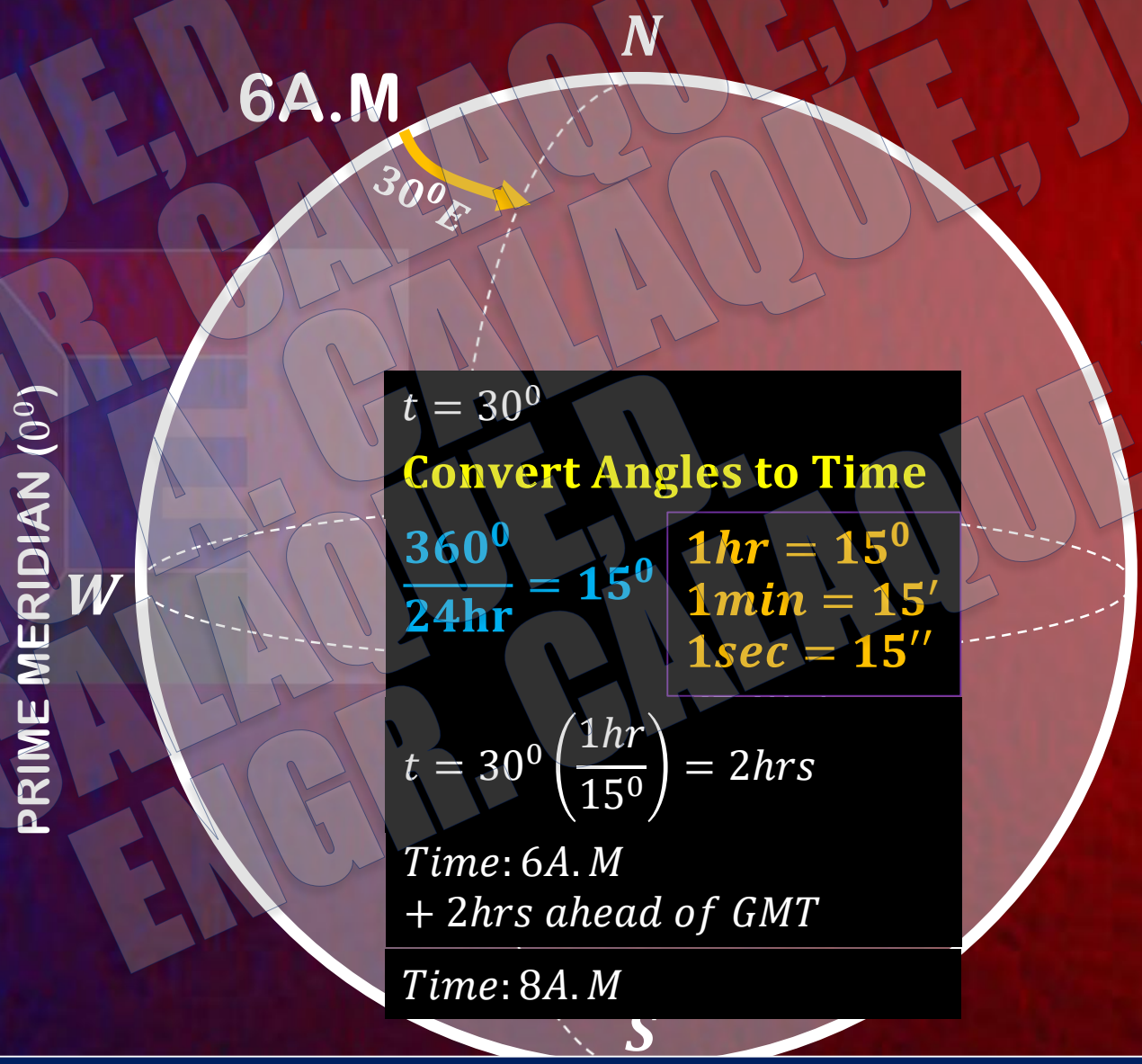
$$\frac{Difference\ in\ Time}{24} = \frac{30^0 - 0^0}{360}$$

$$Difference\ in\ Time = 2hrs$$

$$Time: 6A.M + 2hrs\ ahead\ of\ GMT$$

$$Time: 8A.M$$

SOLUTION:



$t = 30^0$

Convert Angles to Time

$$\frac{360^0}{24hr} = 15^0$$

$1hr = 15^0$
$1min = 15'$
$1sec = 15''$

$$t = 30^0\left(\frac{1hr}{15^0}\right) = 2hrs$$

$Time: 6A.M$
$+ 2hrs\ ahead\ of\ GMT$

$Time: 8A.M$

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PAST CE BOARD EXAM FEB.1977:

FIND THE DISTANCE BETWEEN MANILA ($14^0 36' N, 121^0 05' E$) AND SAN FRANCISCO ($37^0 48^0 N, 122^0 24' W$) AND THEIR BEARINGS FROM EACH OTHER.

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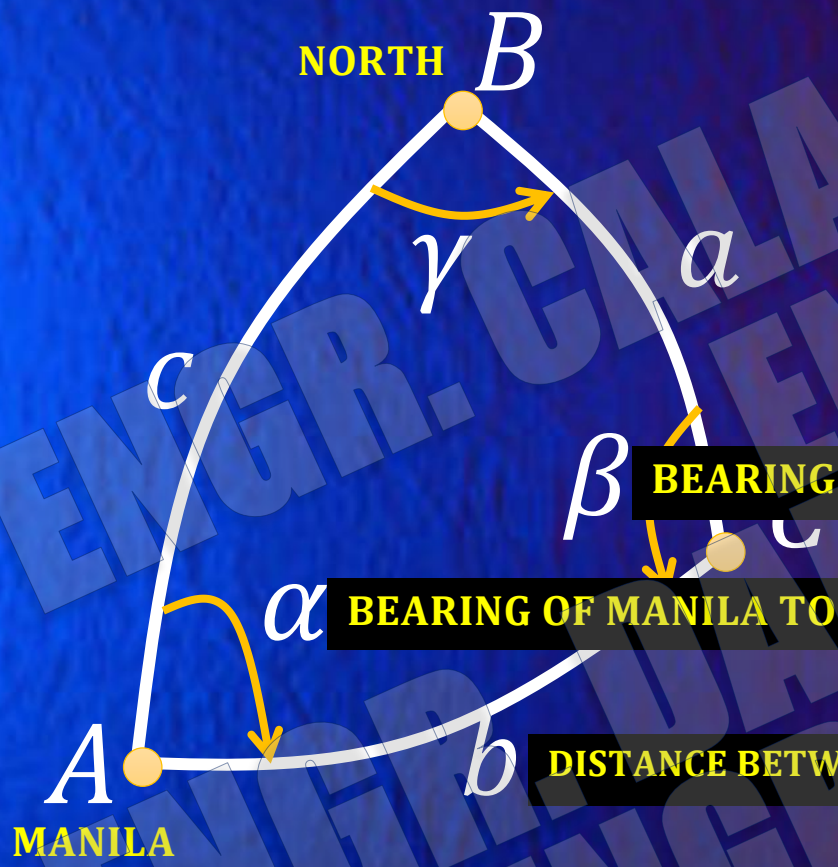
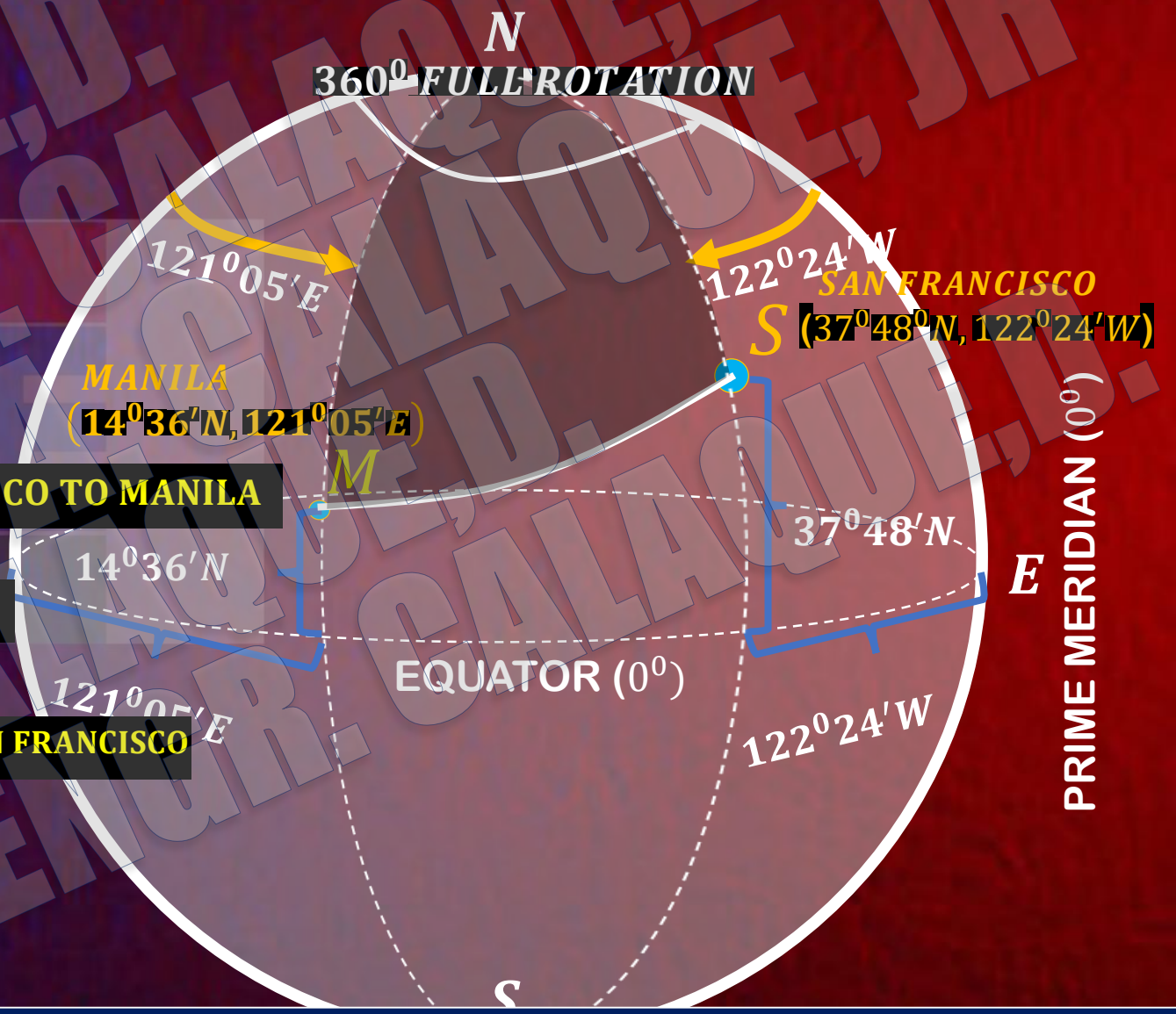


BEARING OF SAN FRANCISCO TO MANILA

BEARING OF MANILA TO SAN FRANCISCO

DISTANCE BETWEEN MANILA & SAN FRANCISCO



$$a = 90^0 - 37^0 48' = 52^0 12'$$

$$c = 90^0 - 14^0 36' = 75^0 24'$$

$$\gamma = 360^0 - 121^0 05' - 122^0 24' = 116^0 31'$$

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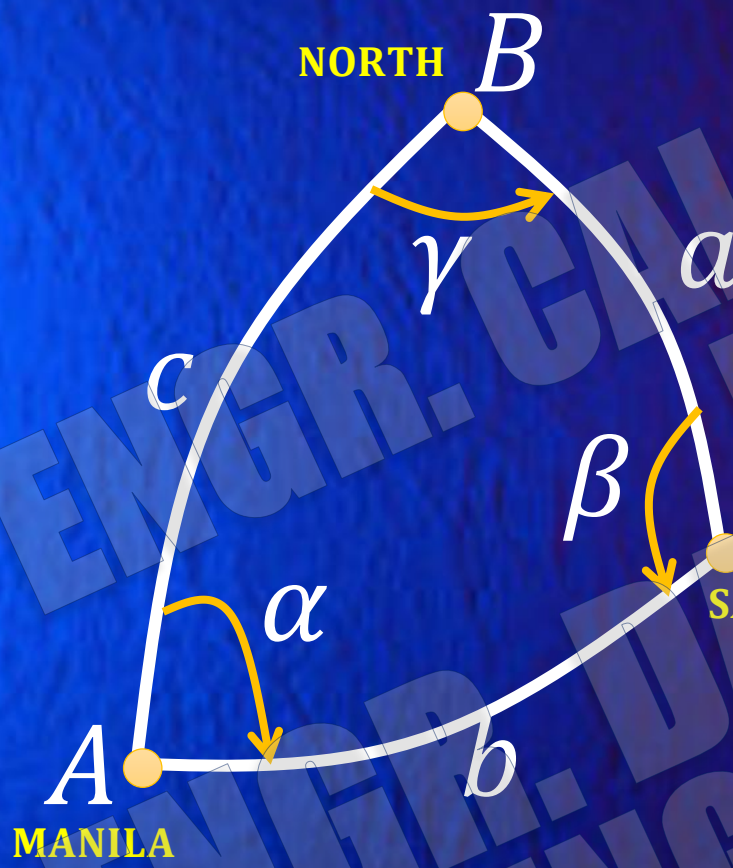
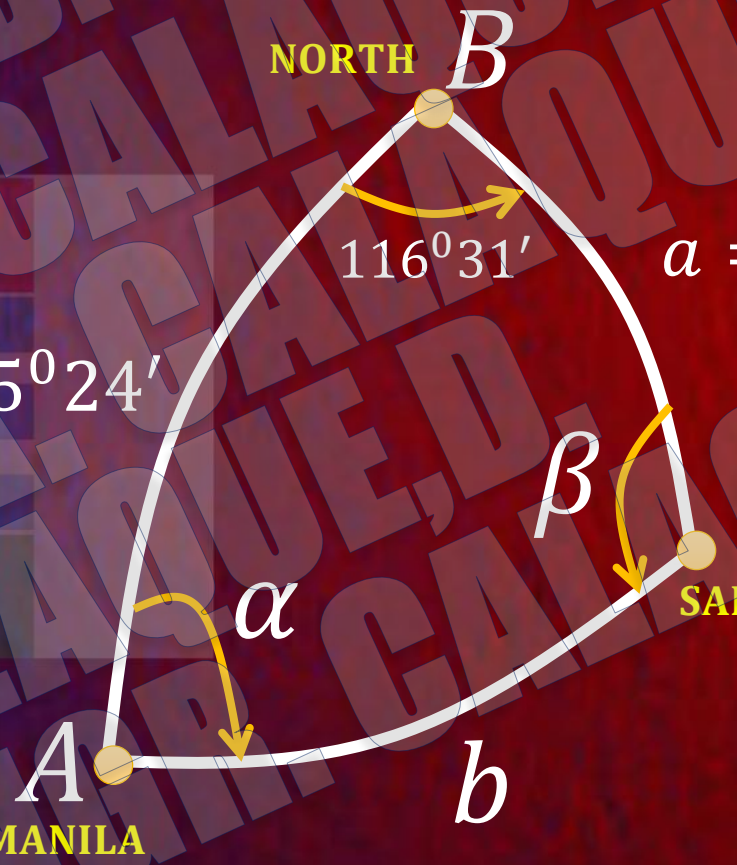


$$a = 90^0 - 37^0 48' = 52^0 12'$$

$$c = 90^0 - 14^0 36' = 75^0 24'$$

$$\gamma = 360^0 - 121^0 05' - 122^0 24' = 116^0 31'$$



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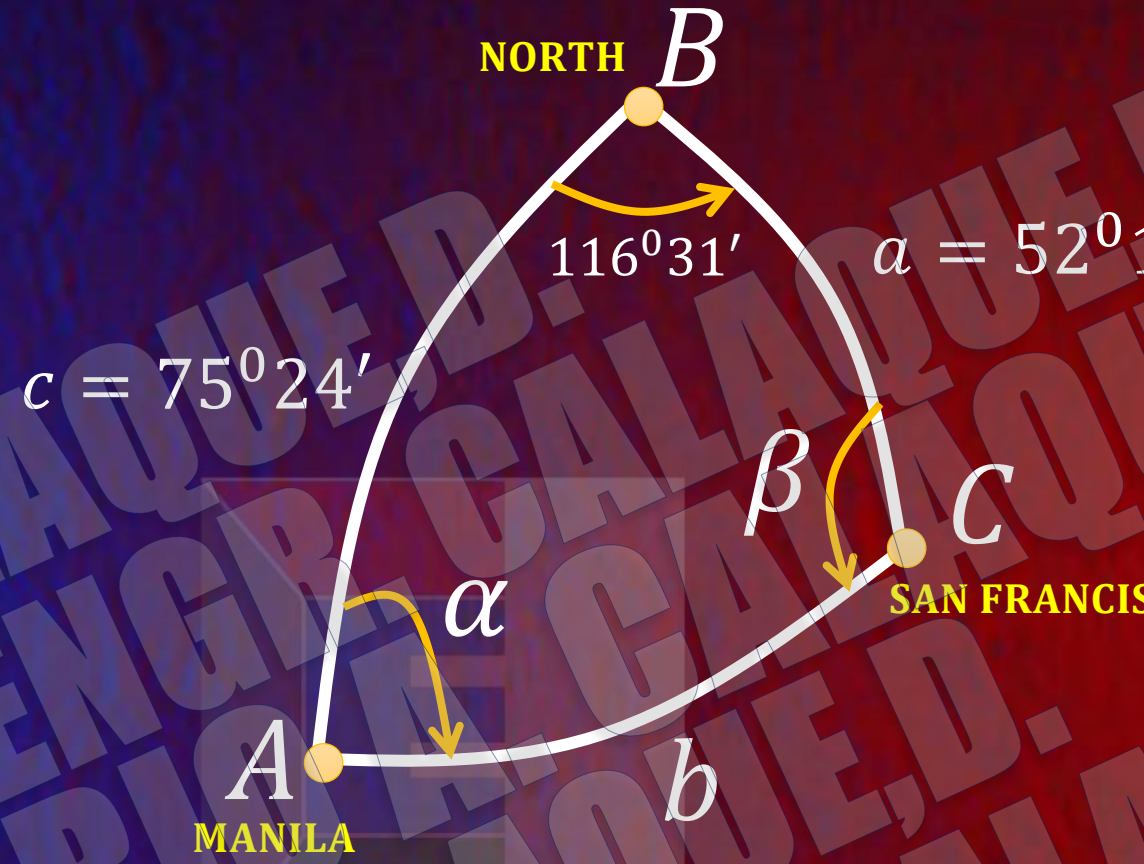

COSINE LAW (FOR THE SIDES):

$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$

$$\cos b = \cos 52^0 12' \cos 75^0 24' + \sin 52^0 12' \sin 75^0 24' \cos 116^0 31'$$

$$b = 100^0 46' 15.74''$$

Note: $1' = 1 n.m$

$$\mathrm{b} = 100^0 \left(\frac{60'}{1^0}\right) + 46' \left(\frac{1nm}{1'}\right) = \mathbf{6046 n.m}$$

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PAST CE BOARD EXAM FEB.1977:

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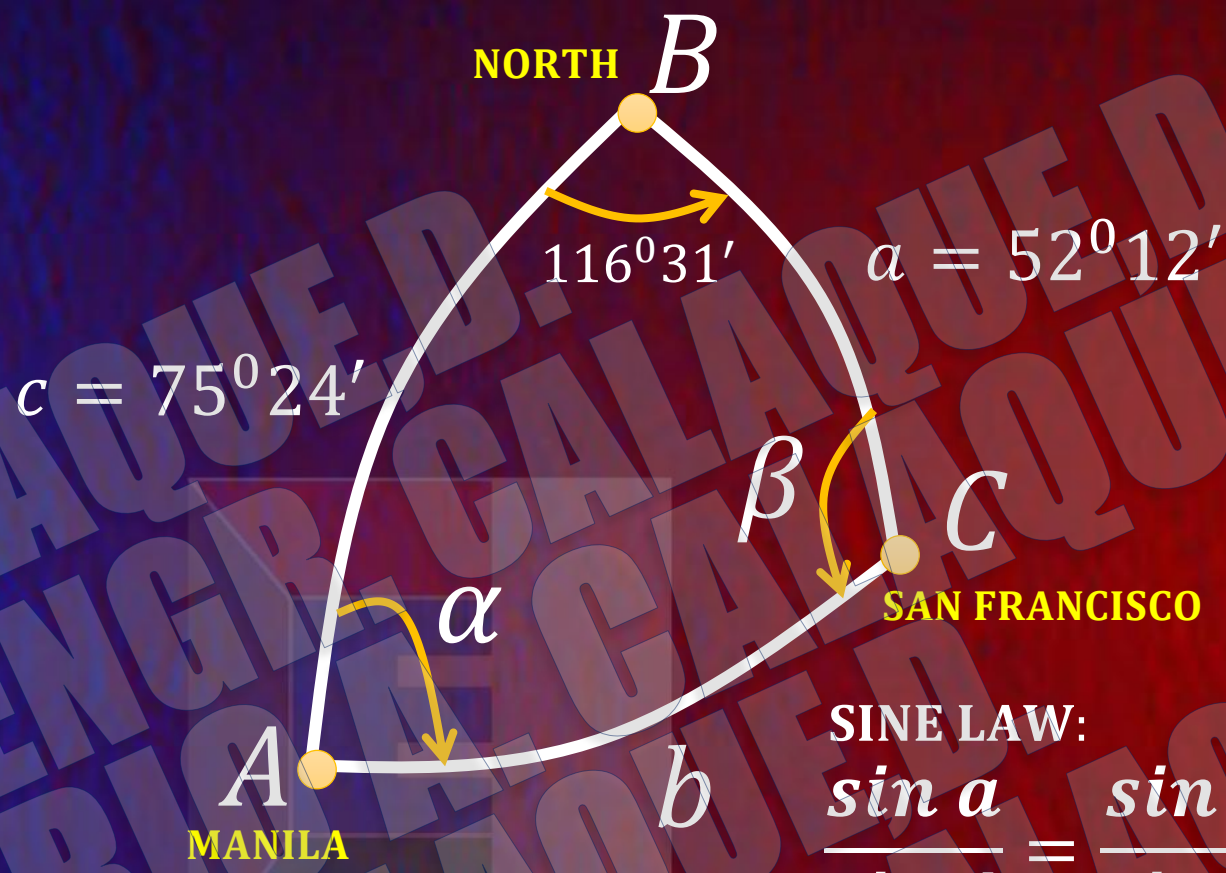

SINE LAW:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

$$\frac{\sin 52^0 12'}{\sin \alpha} = \frac{\sin 100^0 46' 15.74''}{\sin 116^0 31'} = \frac{\sin 75^0 24'}{\sin \beta}$$

BEARING OF MANILA TO SAN FRANCISCO $\alpha = N46^0 01' 51.24'' E$ $\quad \alpha = 46^0 01' 51.24''$

BEARING OF SAN FRANCISCO TO MANILA $\beta = N61^0 48' 59.57'' W$ $\quad \beta = 61^0 48' 59.57''$

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TEST YOURSELF:

Tokyo is located at $(139^0E, 39^0N)$ while Manila is at $(121^0E, 14^0N)$. Find the distance between the two in nautical mile.

A. $1776n.m$

B. $1456n.m$

C. $1467n.m$

D. $1567n.m$

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CELESTIAL SPHERE

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PREPARED BY: ENGR. DARIO A. CALAQUE, JR.

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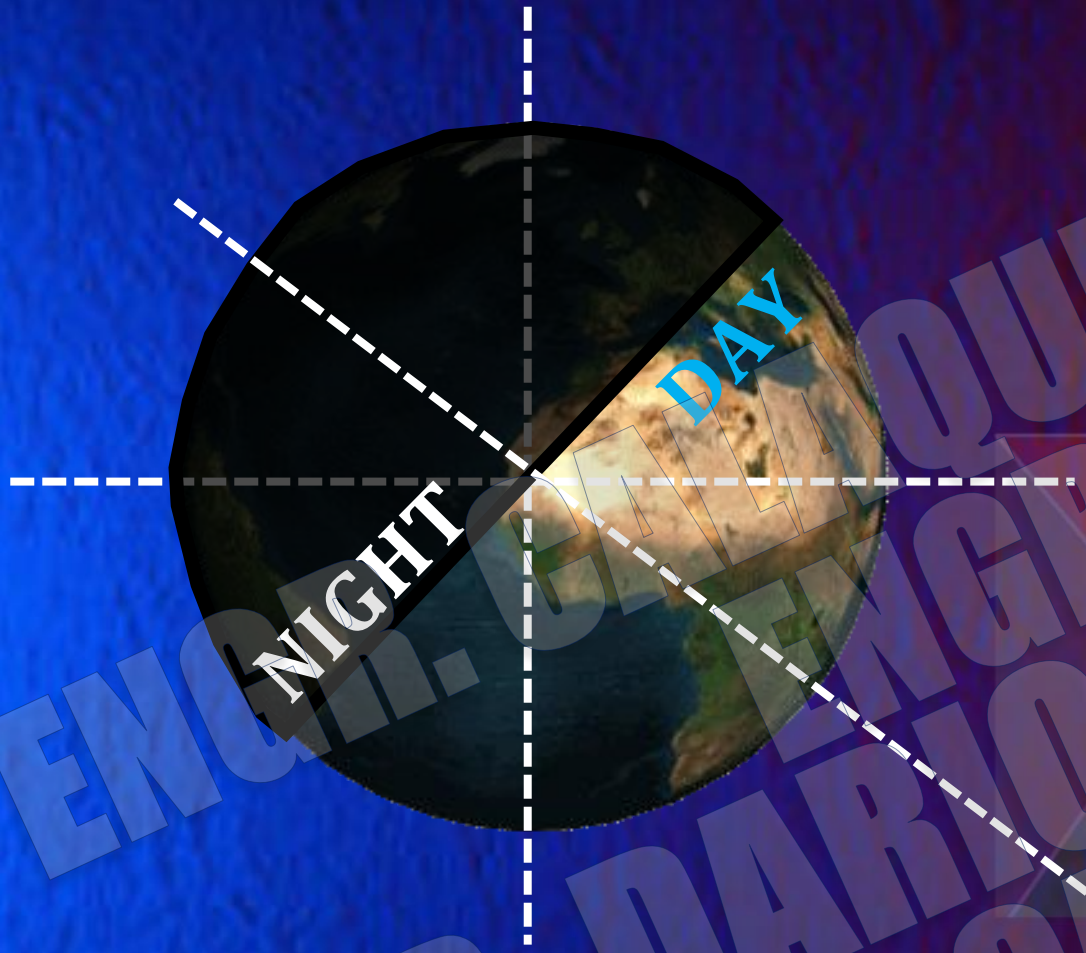

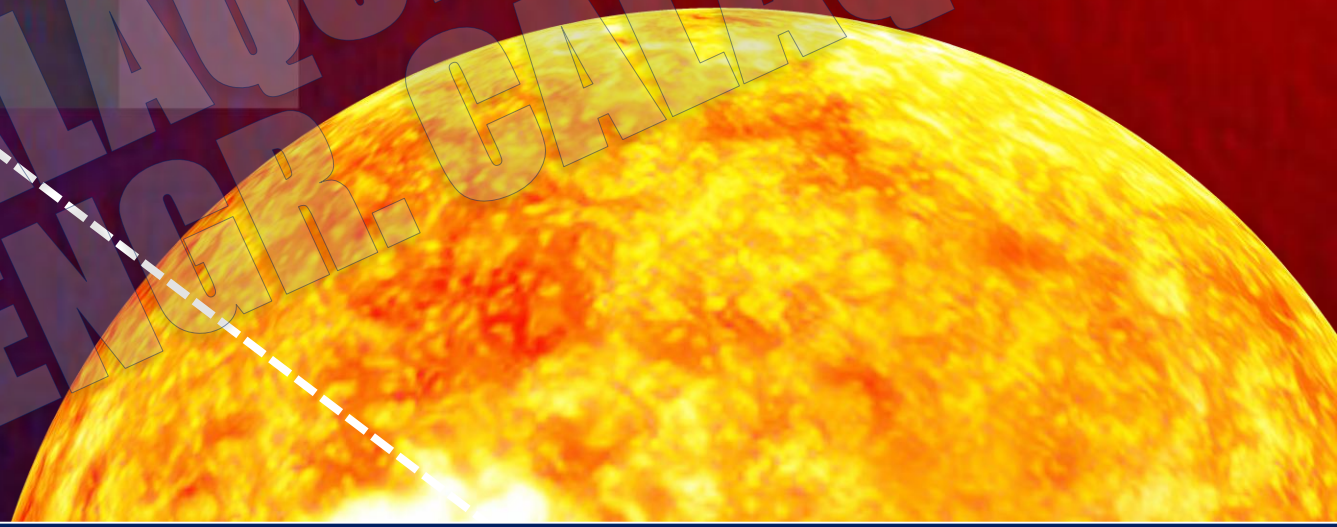

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CELESTIAL SPHERE

Earth's angle of tilt $= 23.5^0$

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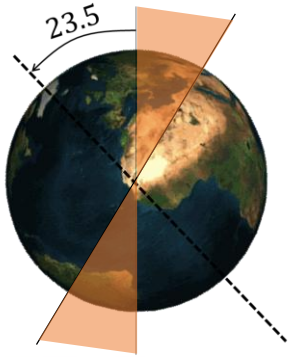

23.5°

North Celestial Pole

Earth's orbit

ECLIPTIC AXIS

ORBIT DIRECTION

South Celestial Pole

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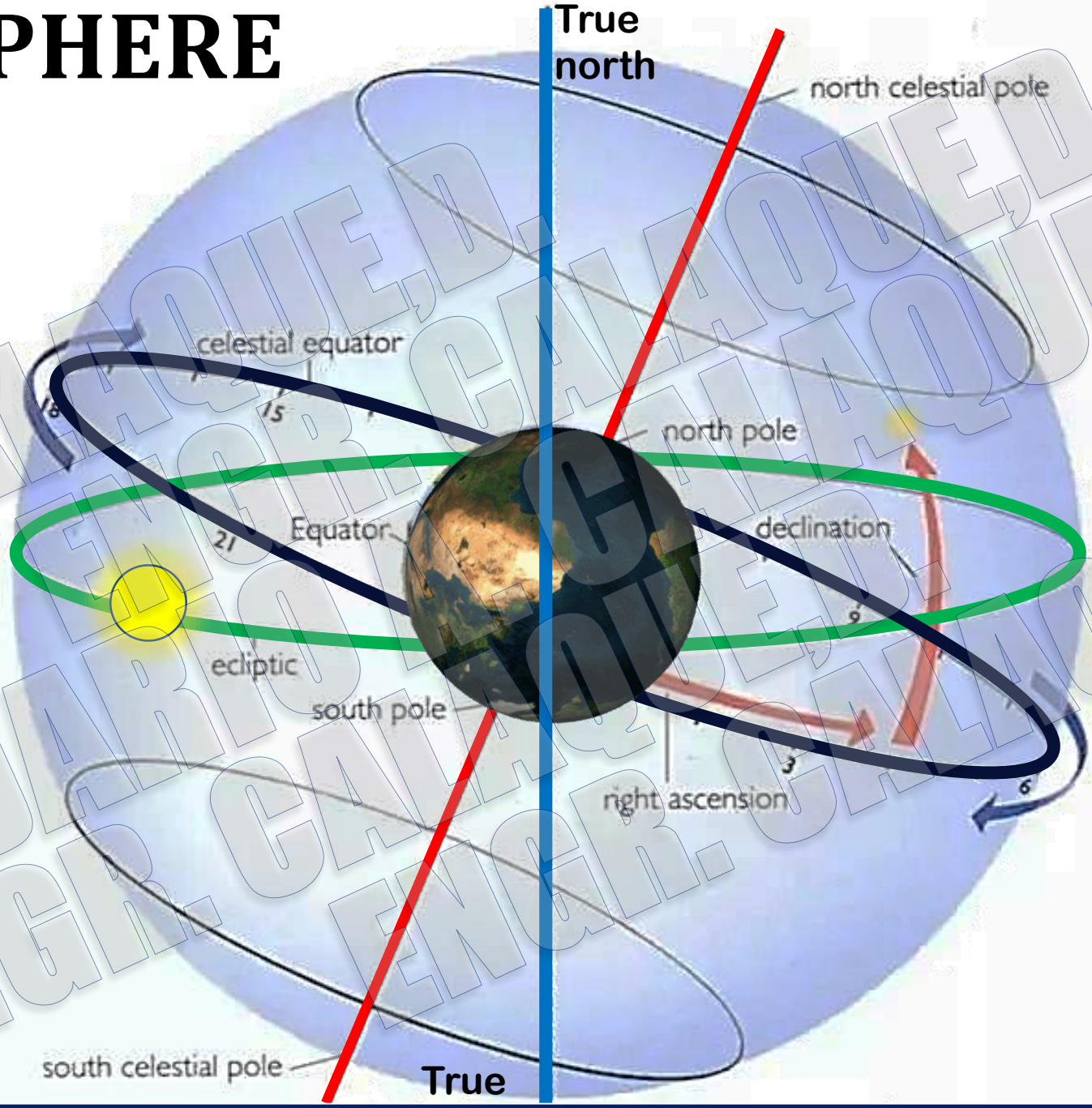
CELESTIAL SPHERE



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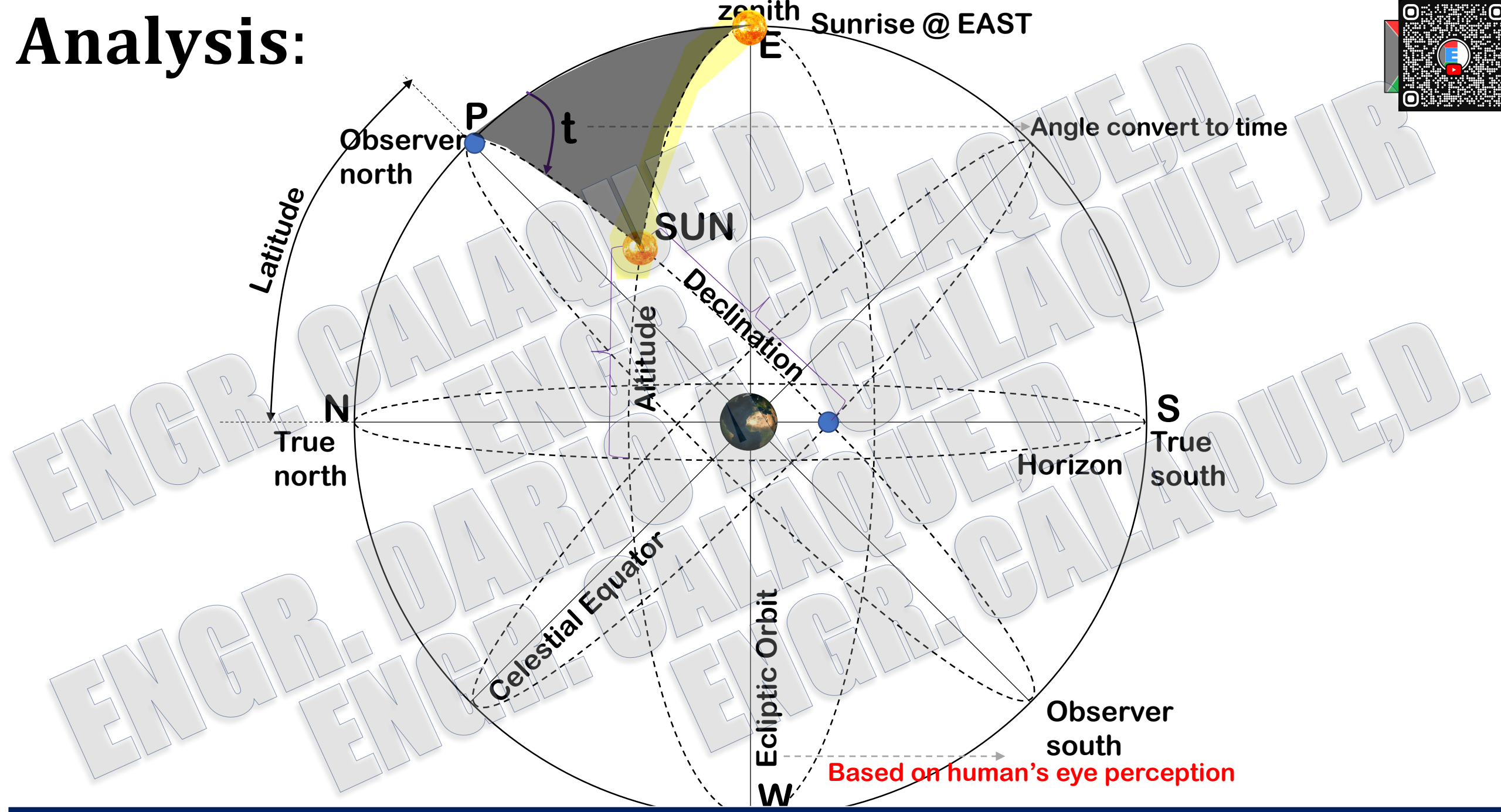
Analysis:



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PAST CE BOARD EXAM FEB.1980:

AN OBSERVER AT LATITUDE $41^0 50' N$ MADE AN OBSERVATION OF THE SUN AND FOUND OUT THE ITS ALTITUDE IS $15^0 40'$. IF THE DECLINATION FOR THAT DAY WAS $12^0 30'$, WHAT WAS THE TIME OF THE DAY IF THE OBSERVATION IS MADE IN THE MORNING?

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PAST CE BOARD EXAM FEB.1980:

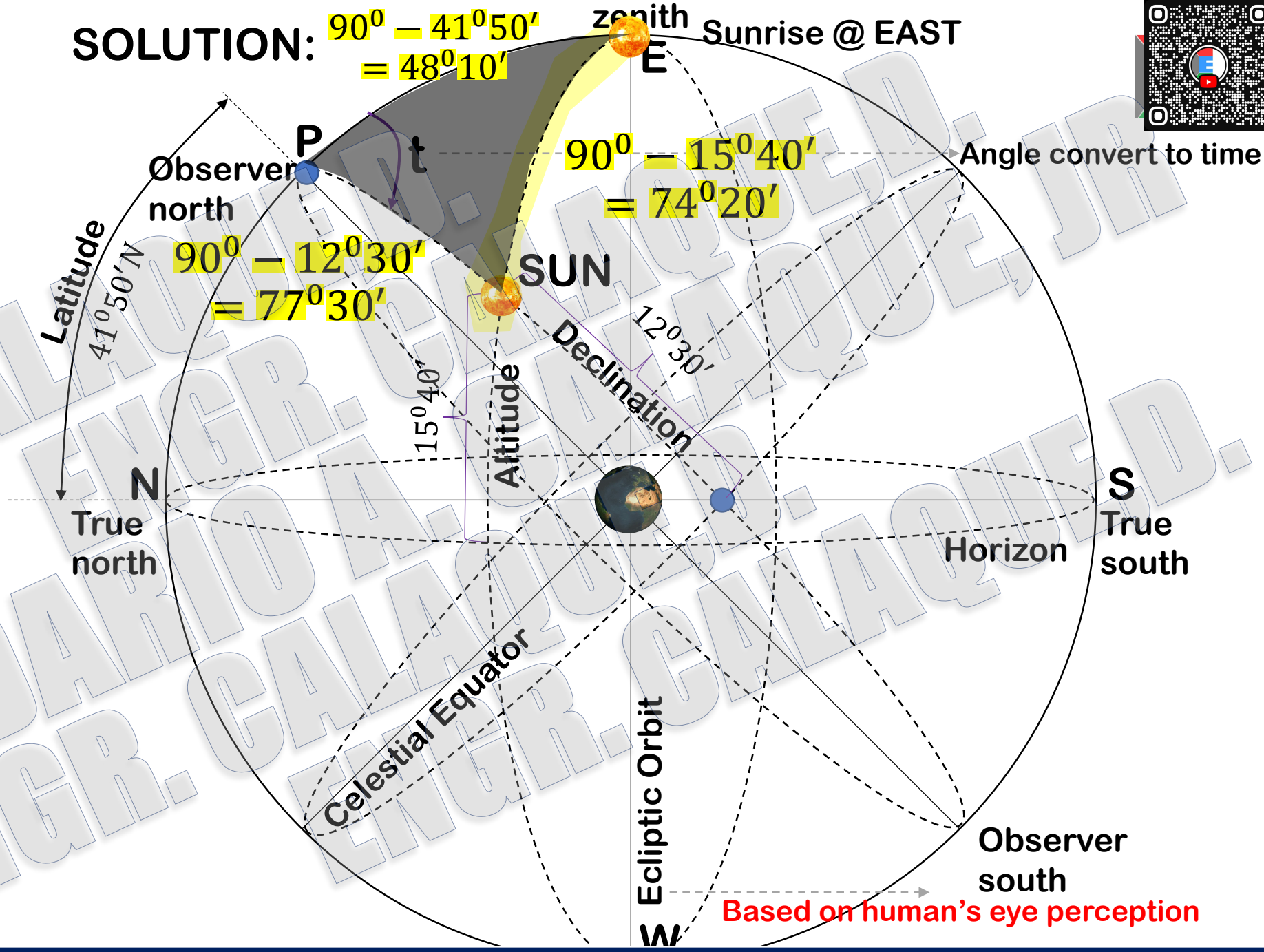
AN OBSERVER AT LATITUDE $41^0 50' N$ MADE AN OBSERVATION OF THE SUN AND FOUND OUT THE ITS ALTITUDE IS $15^0 40'$. IF THE DECLINATION FOR THAT DAY WAS $12^0 30'$, WHAT WAS THE TIME OF THE DAY IF THE OBSERVATION IS MADE IN THE MORNING?

SOLUTION:

$90^0 - 41^0 50'$
$= 48^0 10'$

$90^0 - 15^0 40'$
$= 74^0 20'$

$90^0 - 12^0 30'$
$= 77^0 30'$



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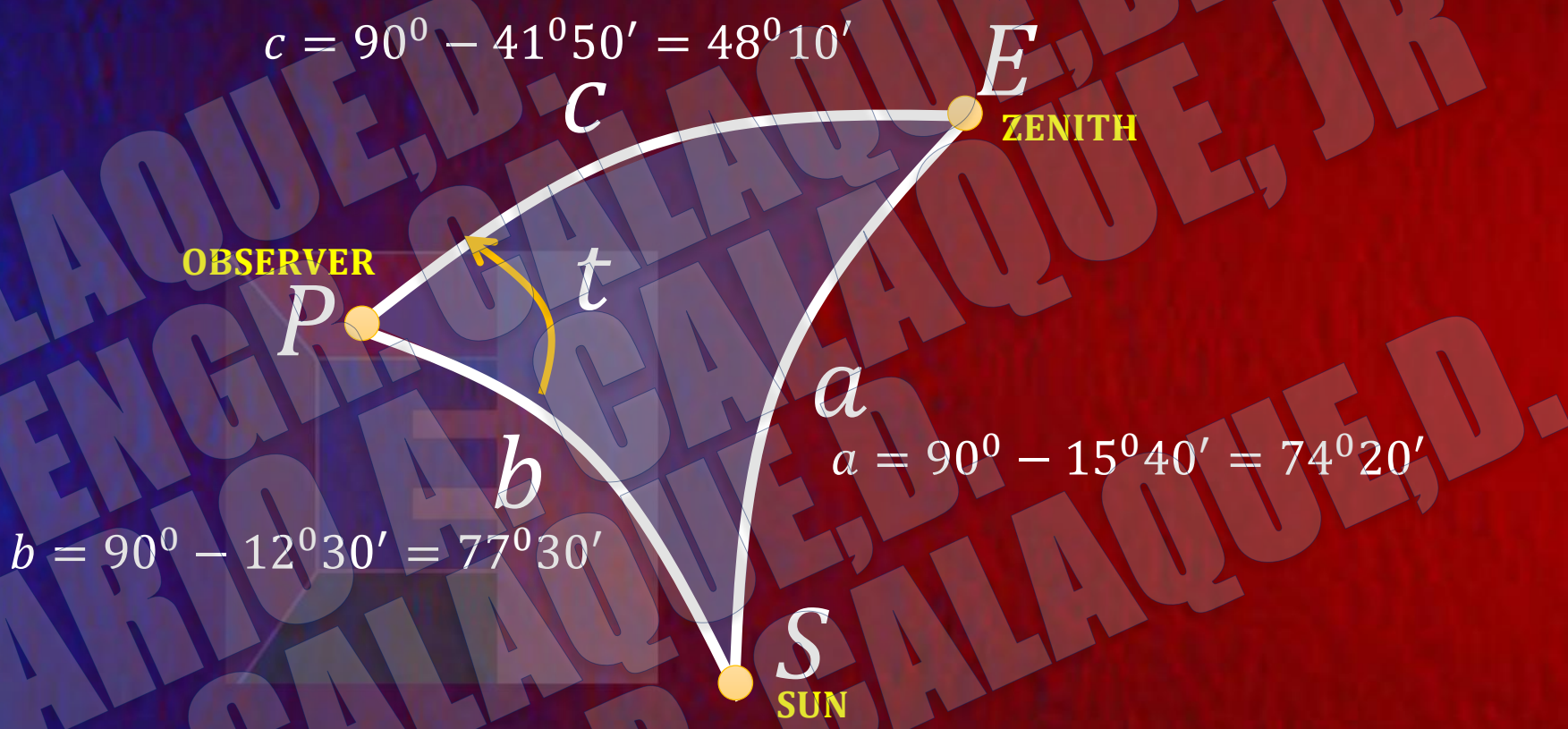
PAST CE BOARD EXAM FEB.1980:

AN OBSERVER AT LATITUDE $41^0 50' N$ MADE AN OBSERVATION OF THE SUN AND FOUND OUT THE ITS ALTITUDE IS $15^0 40'$. IF THE DECLINATION FOR THAT DAY WAS $12^0 30'$, WHAT WAS THE TIME OF THE DAY IF THE OBSERVATION IS MADE IN THE MORNING?

SOLUTION:



COSINE LAW (FOR THE SIDES):

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos 74^0 20' = \cos 77^0 30' \cos 48^0 10' + \sin 77^0 30' \sin 48^0 10' \cos t$$

$$t = 80.05069167^0$$

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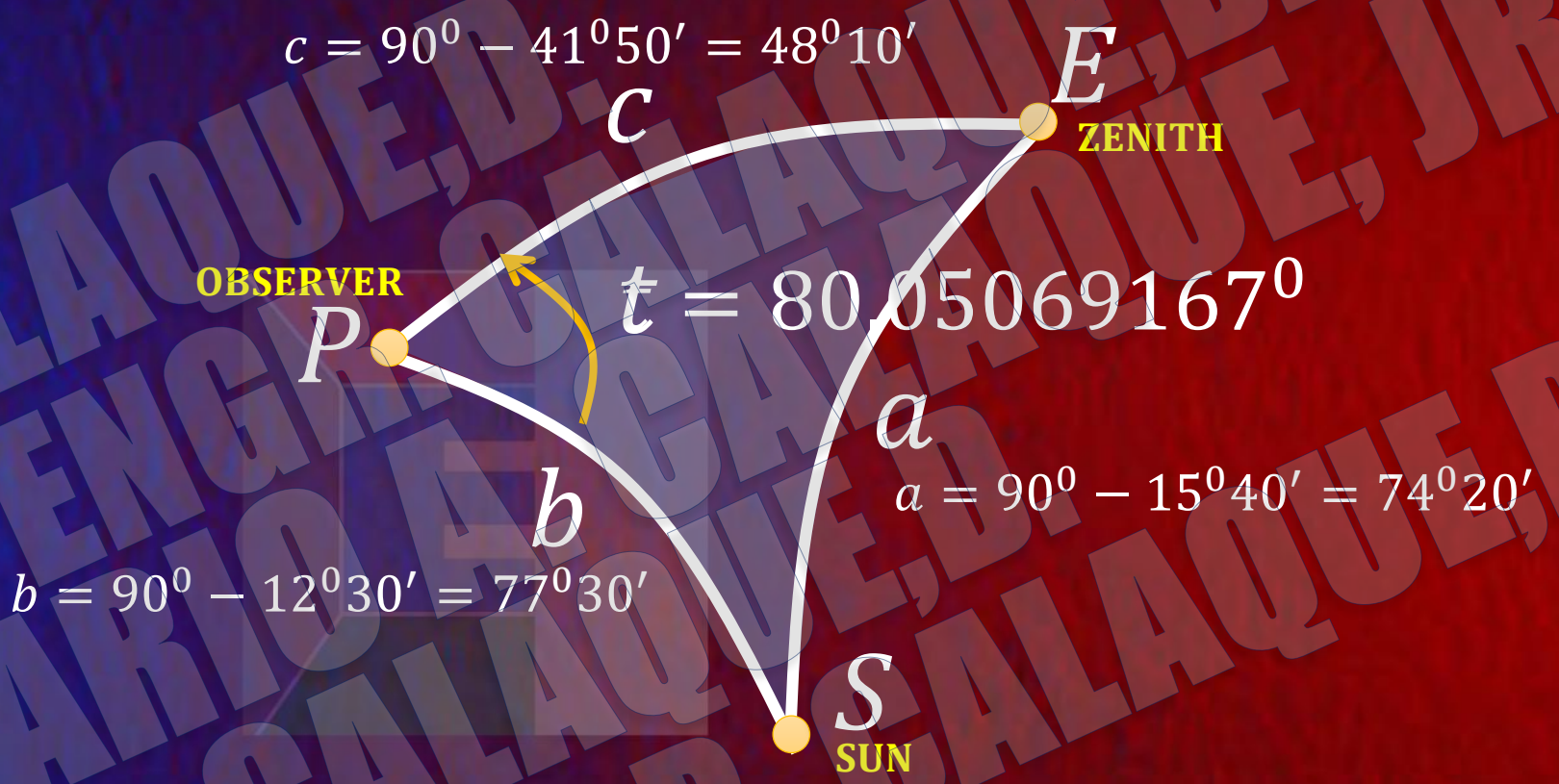
PAST CE BOARD EXAM FEB.1980:

AN OBSERVER AT LATITUDE $41^0 50' N$ MADE AN OBSERVATION OF THE SUN AND FOUND OUT THE ITS ALTITUDE IS $15^0 40'$. IF THE DECLINATION FOR THAT DAY WAS $12^0 30'$, WHAT WAS THE TIME OF THE DAY IF THE OBSERVATION IS MADE IN THE MORNING?

SOLUTION:



Convert Angles to Time

$$\frac{360^0}{24hr} = 15^0$$

$$t = 80.05069167^0 \left(\frac{1}{15^0}\right)$$

$$t = 5^0 20' 12.17''$$

$$t = 5hr\,20\,\text{min}\,12.17sec$$

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PAST CE BOARD EXAM FEB.1980:

AN OBSERVER AT LATITUDE $41^0 50' N$ MADE AN OBSERVATION OF THE SUN AND FOUND OUT THE ITS ALTITUDE IS $15^0 40'$. IF THE DECLINATION FOR THAT DAY WAS $12^0 30'$, WHAT WAS THE TIME OF THE DAY IF THE OBSERVATION IS MADE IN THE MORNING?

SOLUTION:

TIME OF OBSERVATION:

$$12:00:00NN - 5:20:12.17 = \boxed{6:39:47.83AM}$$

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TEST YOURSELF:

The sun was observed in the eastern sky from Manila $(121^0 E\ long. 14^0 N)$. By using sextant, it was found that the sun has a declination of 50^0N. If the altitude of the sun is 40^0, find the local apparent time.

A. 9:09

B. 11:12

C. 10:36

D. 2:51

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PLANE GEOMETRY

ENGR. DARIO A. CALAQUE, JR.

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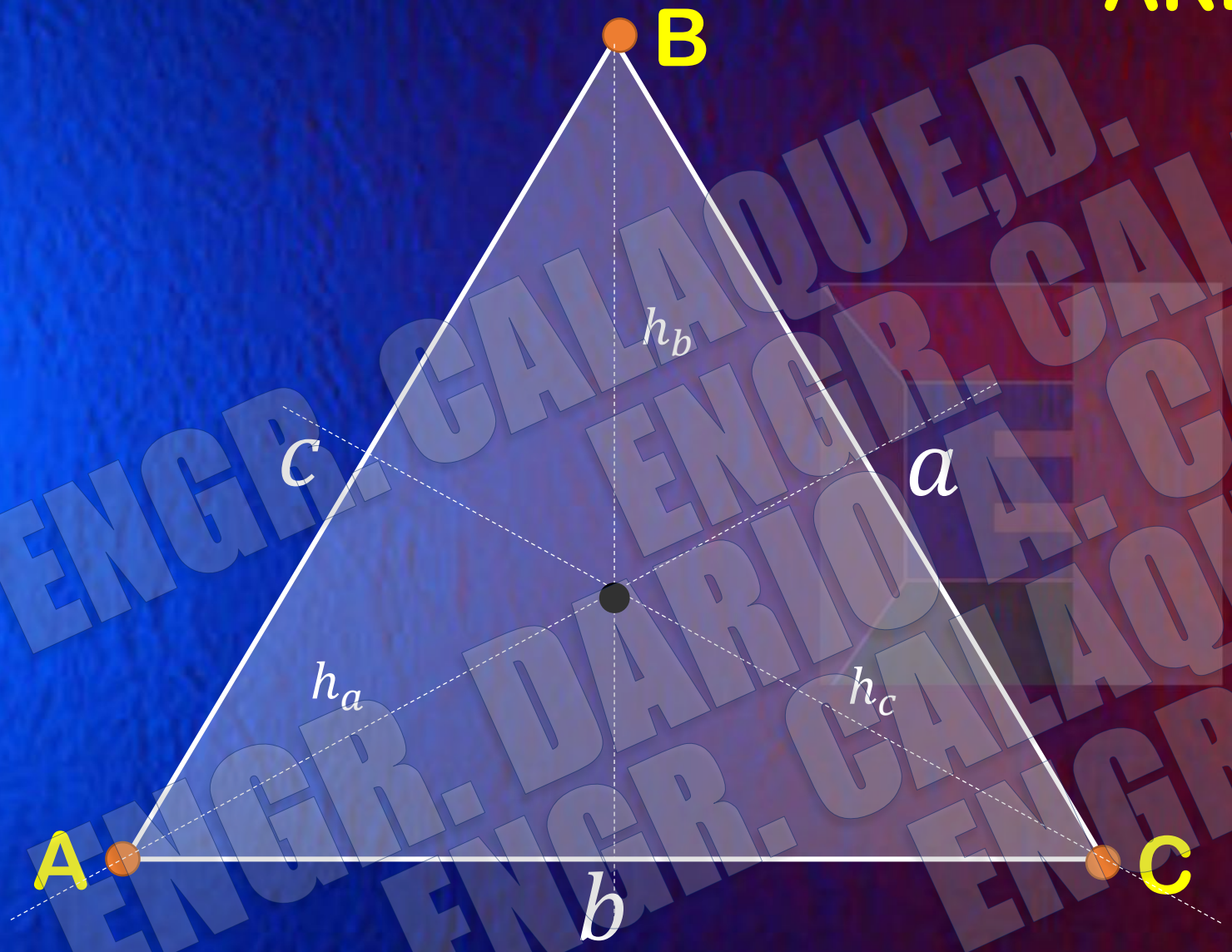
AREA OF TRIANGLE



$$A = \frac{1}{2} b(h_b)$$

$$A = \frac{1}{2} b\, c \sin A$$

$$A = \frac{1}{2}\left(\frac{b^2}{\sin B}\right) sinA \sin C$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

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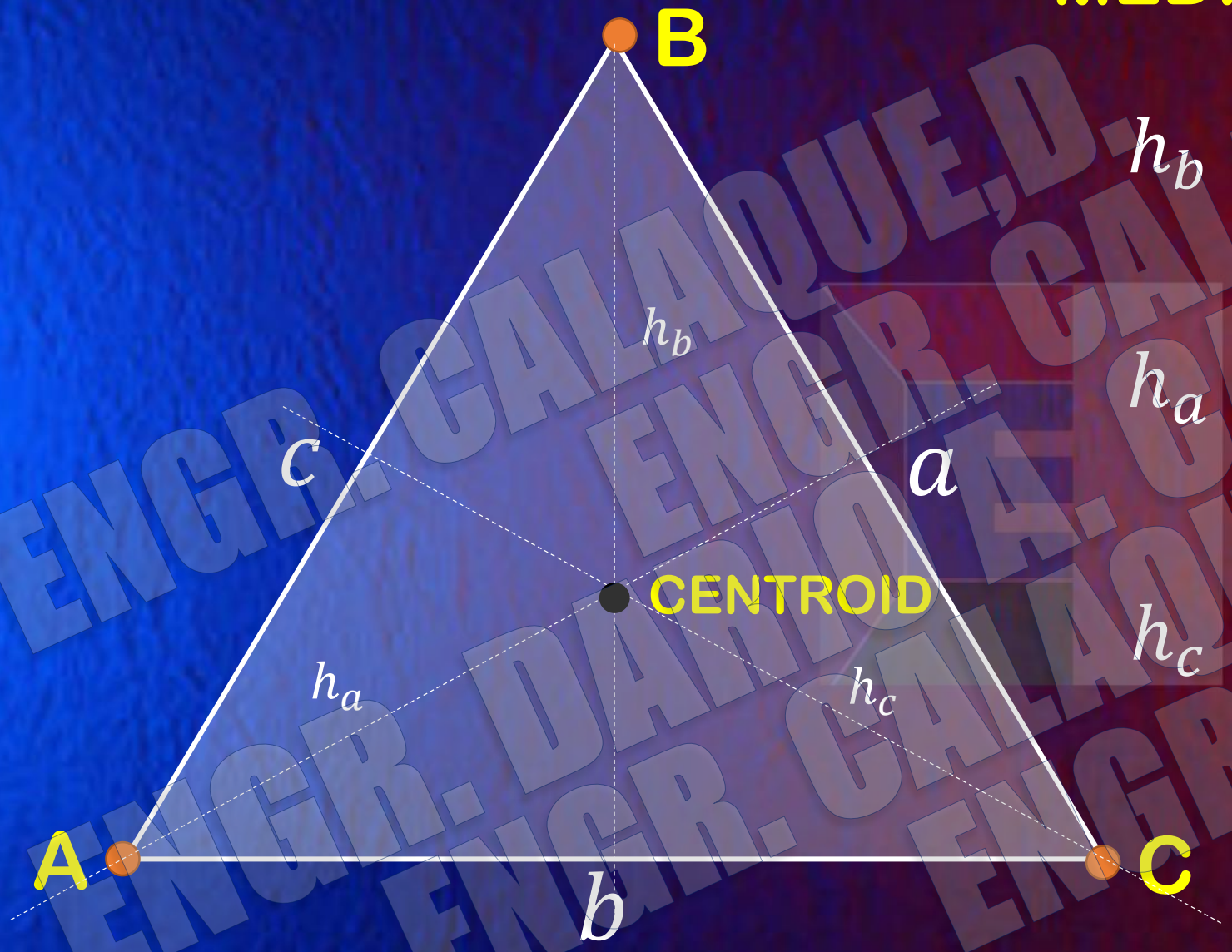
MEDIANS OF TRIANGLE



$$h_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$$

$$h_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

$$h_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$$

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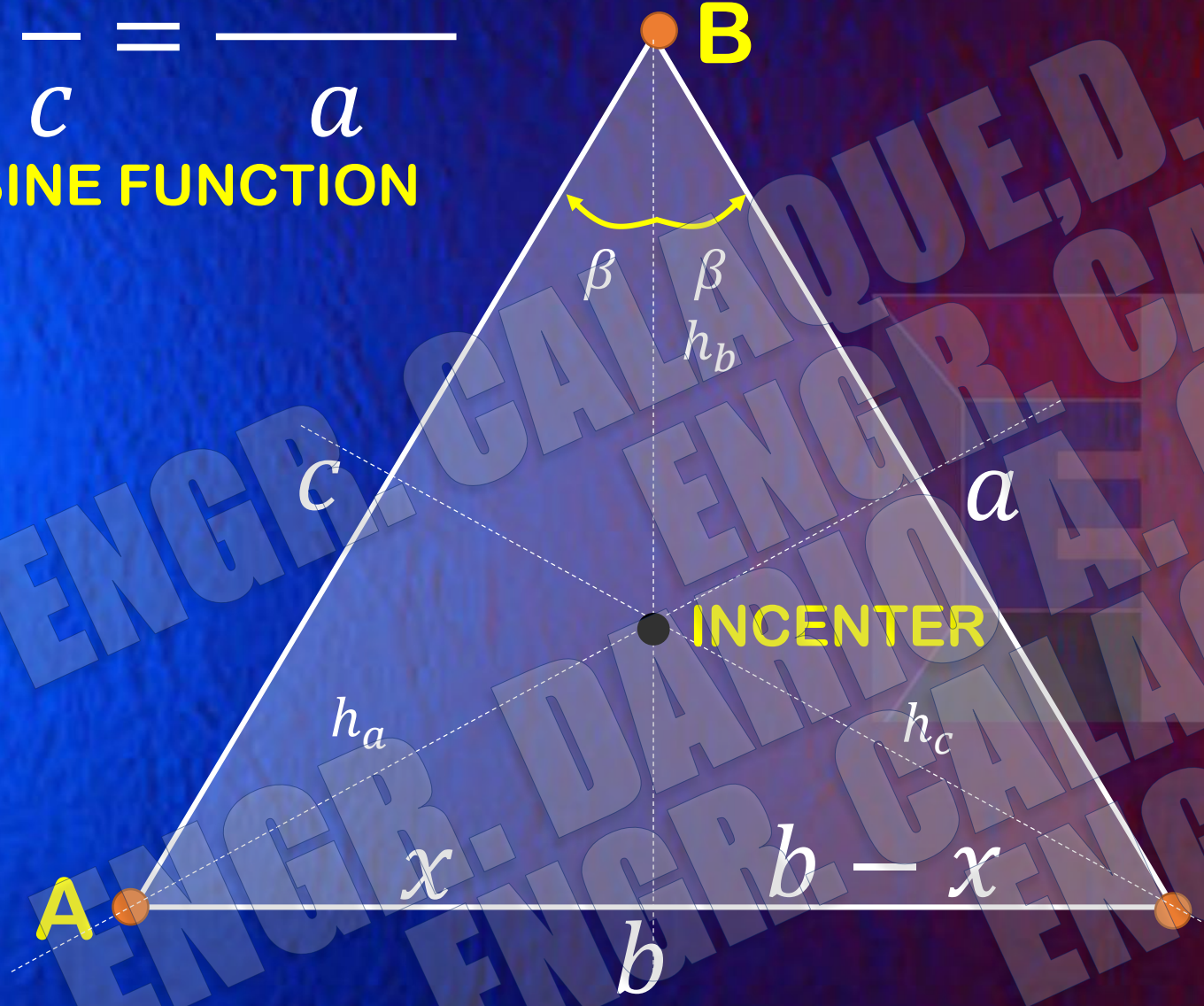
ANGLE BISECTOR OF TRIANGLE

$$\frac{x}{c} = \frac{b-x}{a}$$

SINE FUNCTION



$$h_a = \frac{2}{b+c}\sqrt{bcs(s-a)}$$

$$h_b = \frac{2}{a+c}\sqrt{acs(s-b)}$$

$$h_c = \frac{2}{a+b}\sqrt{abs(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

$$(h_x)^2 = ac - x(b-x)$$

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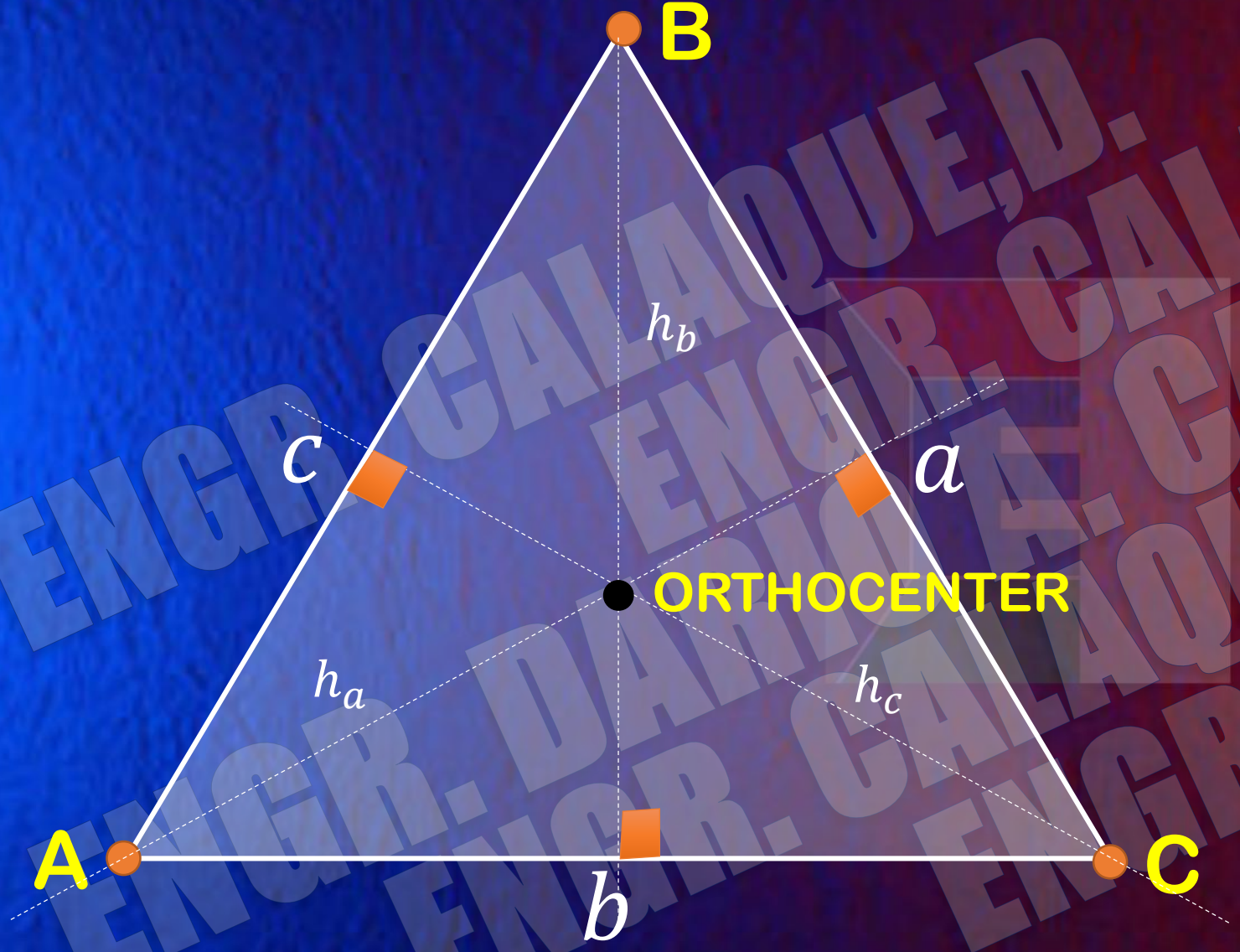
ALTITUDES OF TRIANGLE



$$h_a = \frac{2A_T}{a}$$

$$h_b = \frac{2A_T}{b}$$

$$h_c = \frac{2A_T}{c}$$

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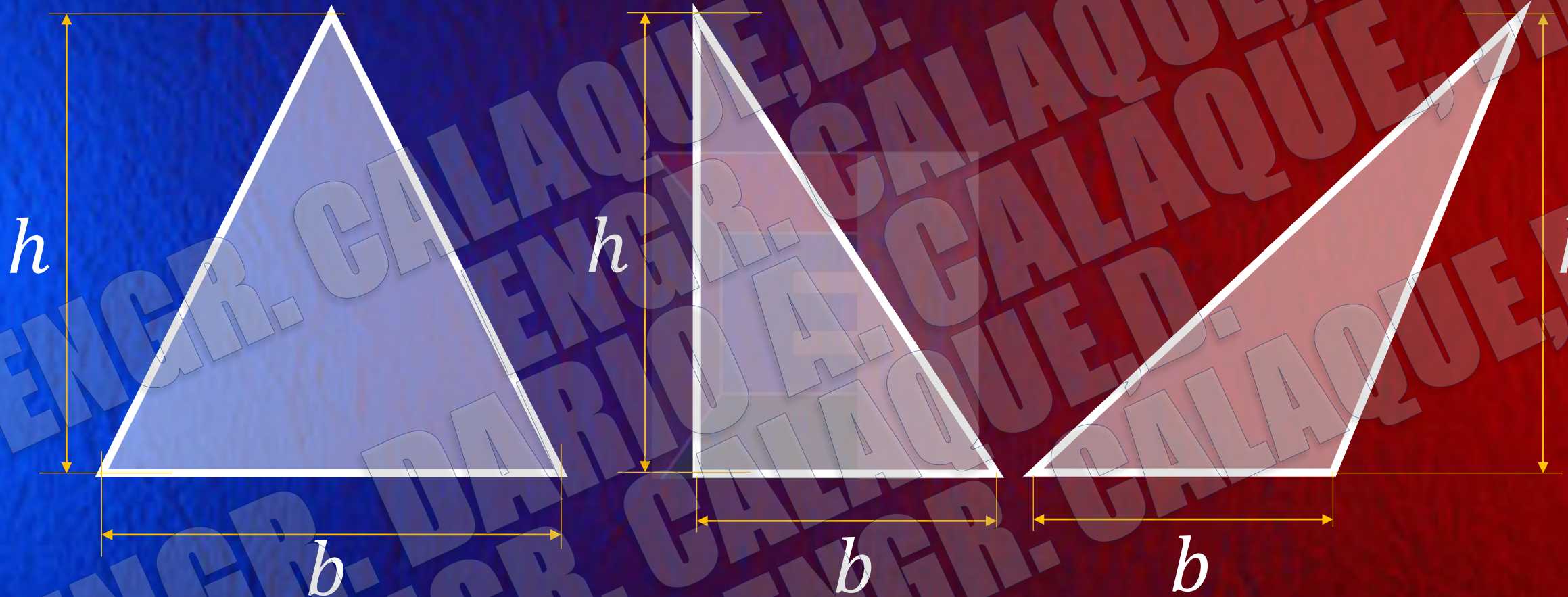
CAVALIERI'S THEOREM



$$A = \frac{1}{2} b(h)$$

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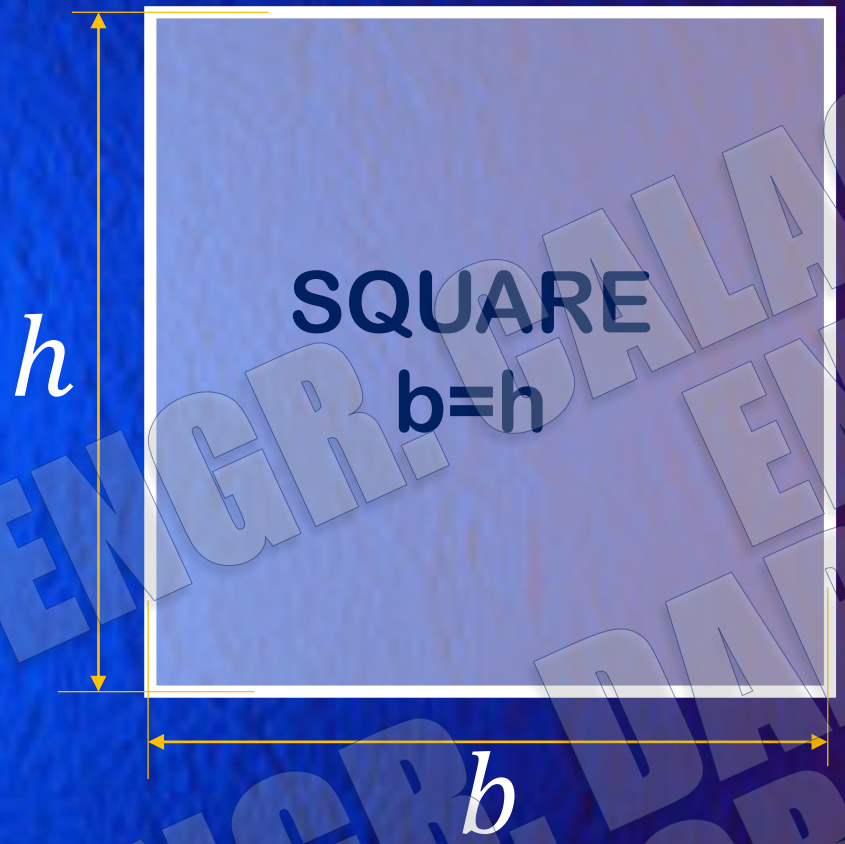
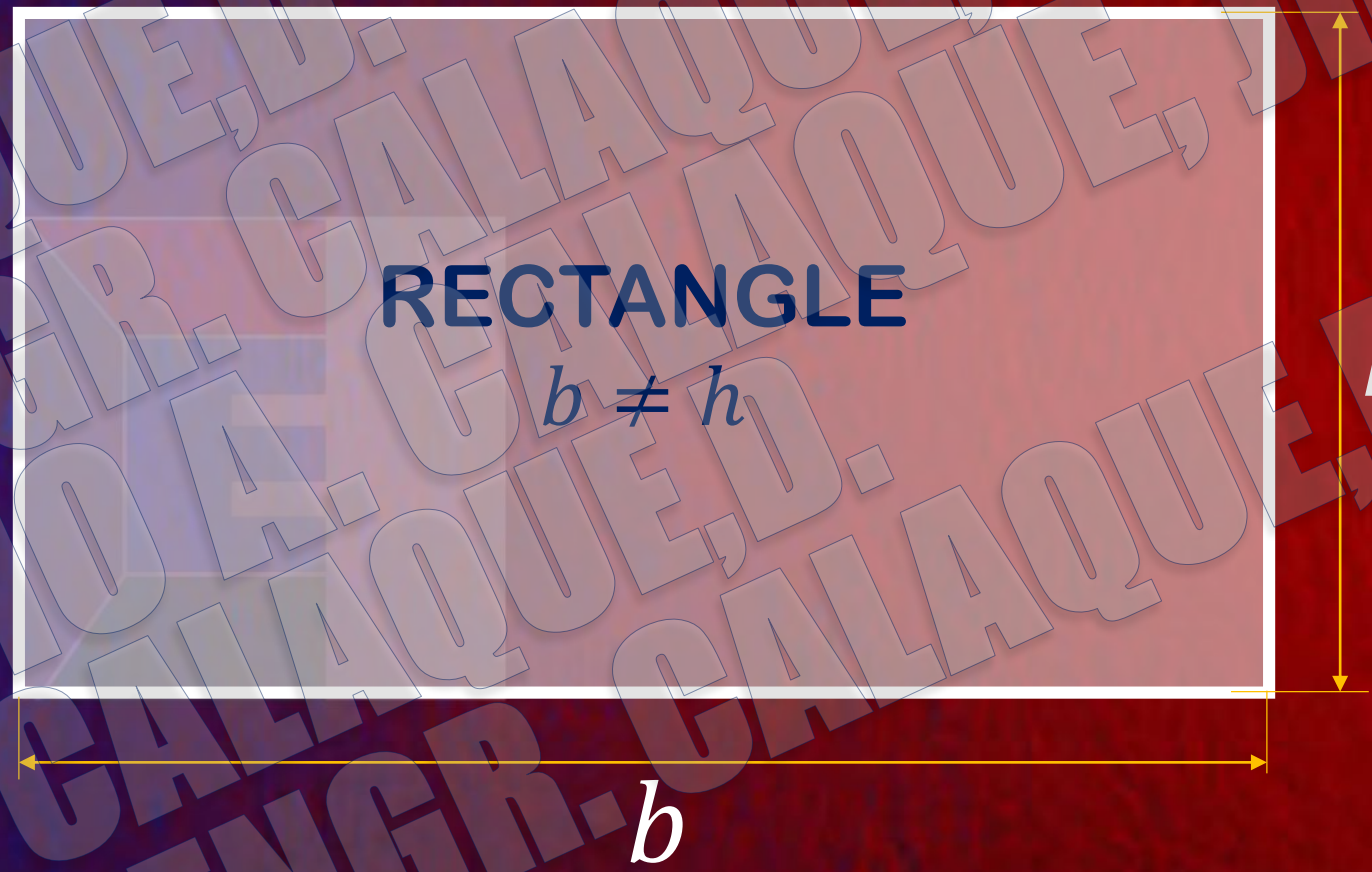
SQUARE AND RECTANGLE





$$A = bh$$

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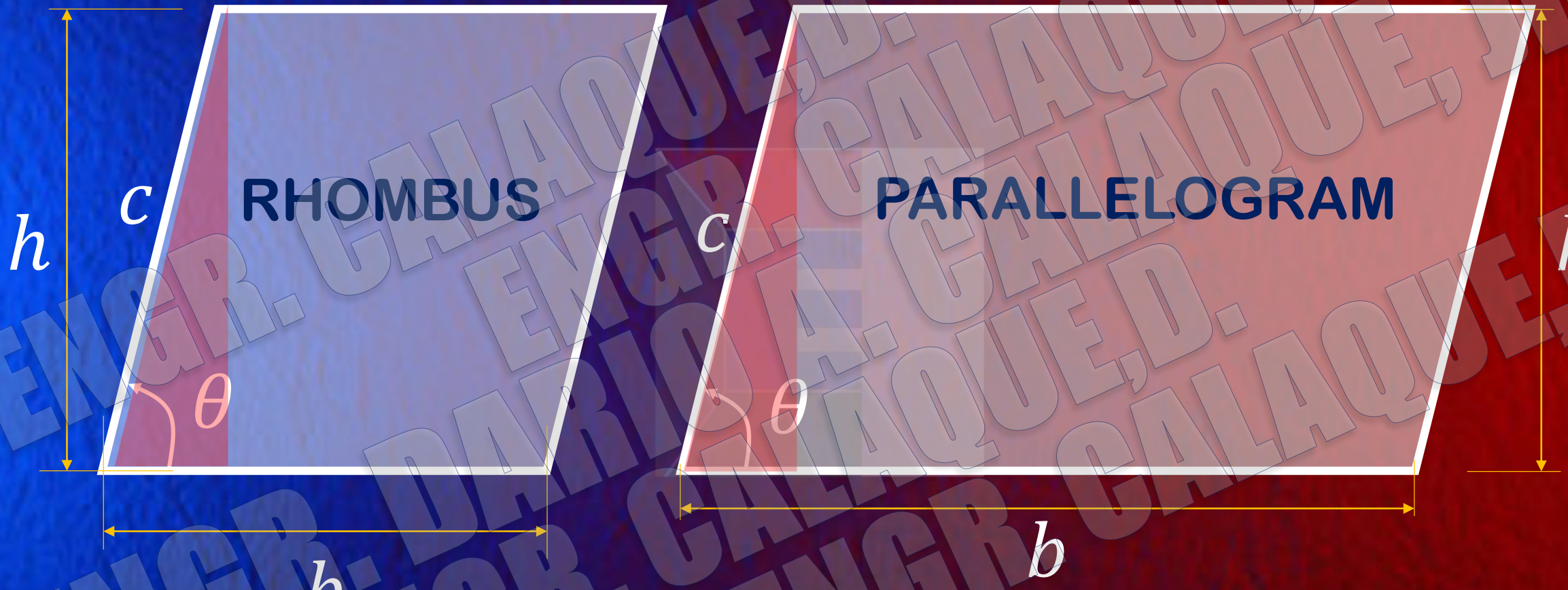
RHOMBUS AND PARALLELOGRAM



$A = bh$	$h = c \sin\theta$
$A = bc \sin\theta$	

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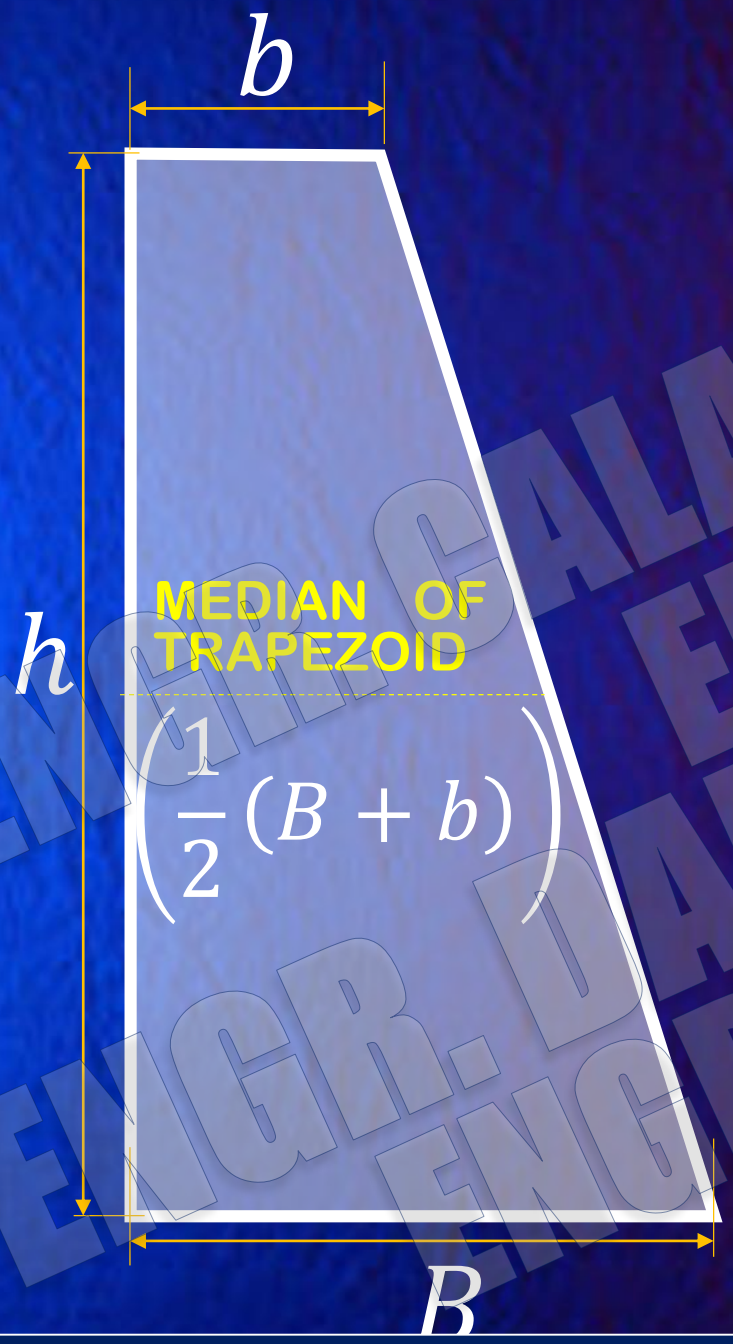
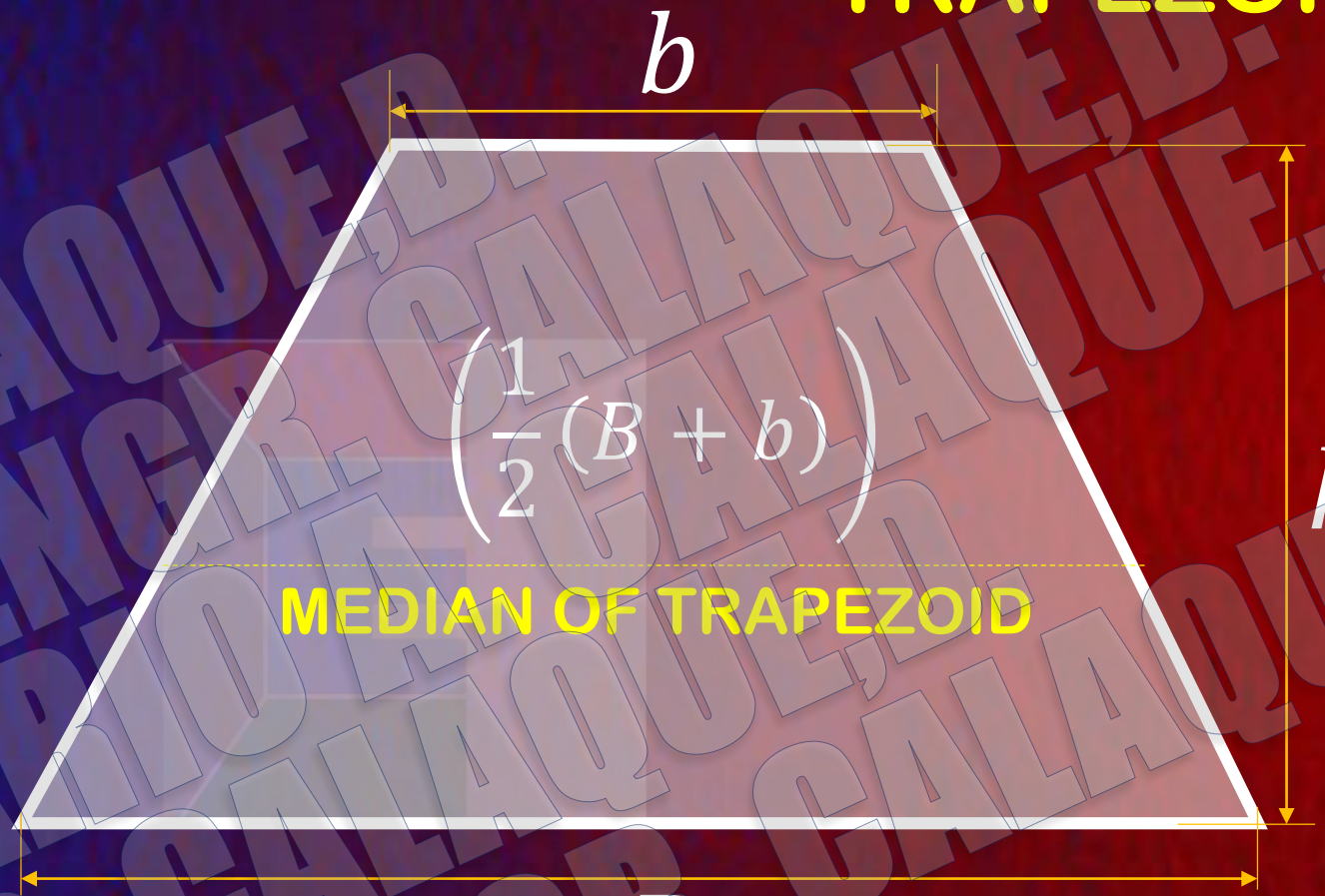
TRAPEZOIDS





$$A = b_{average}\, h$$

$$A = \left(\frac{1}{2}(B+b)\right) h$$

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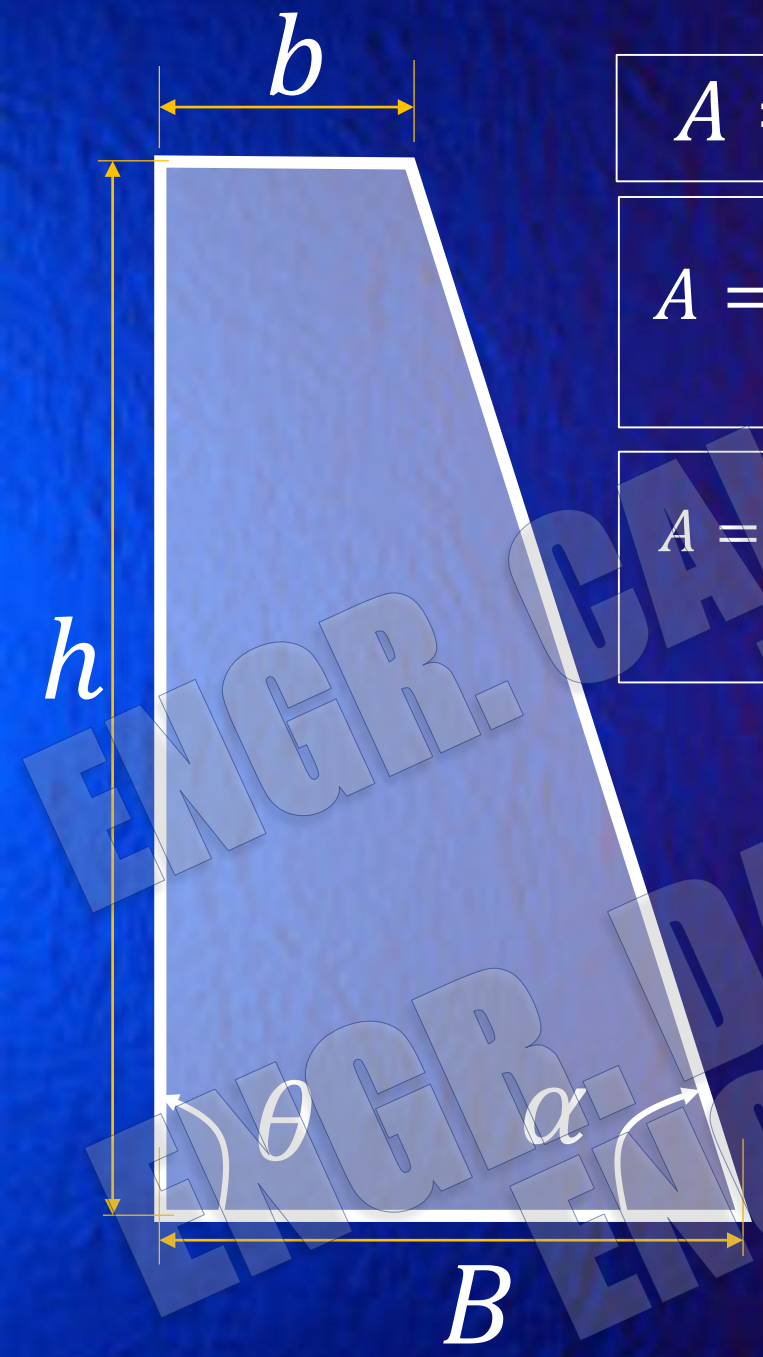
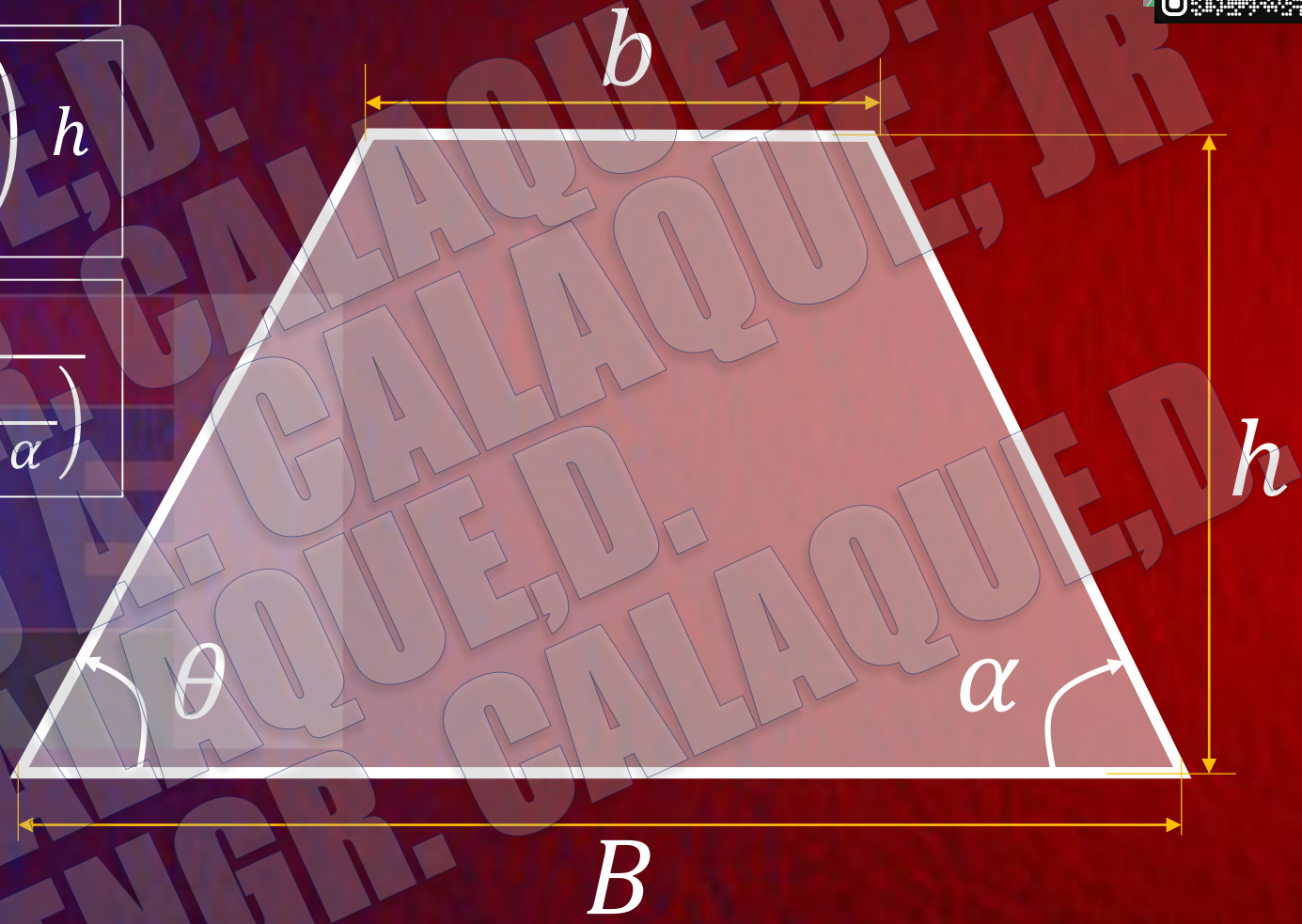
TRAPEZOIDS



$$A = b_{average}\, h$$

$$A = \left(\frac{1}{2}(B + b)\right) h$$

$$A = \frac{B^2 - b^2}{2\left(\frac{1}{\tan\theta} + \frac{1}{\tan\alpha}\right)}$$



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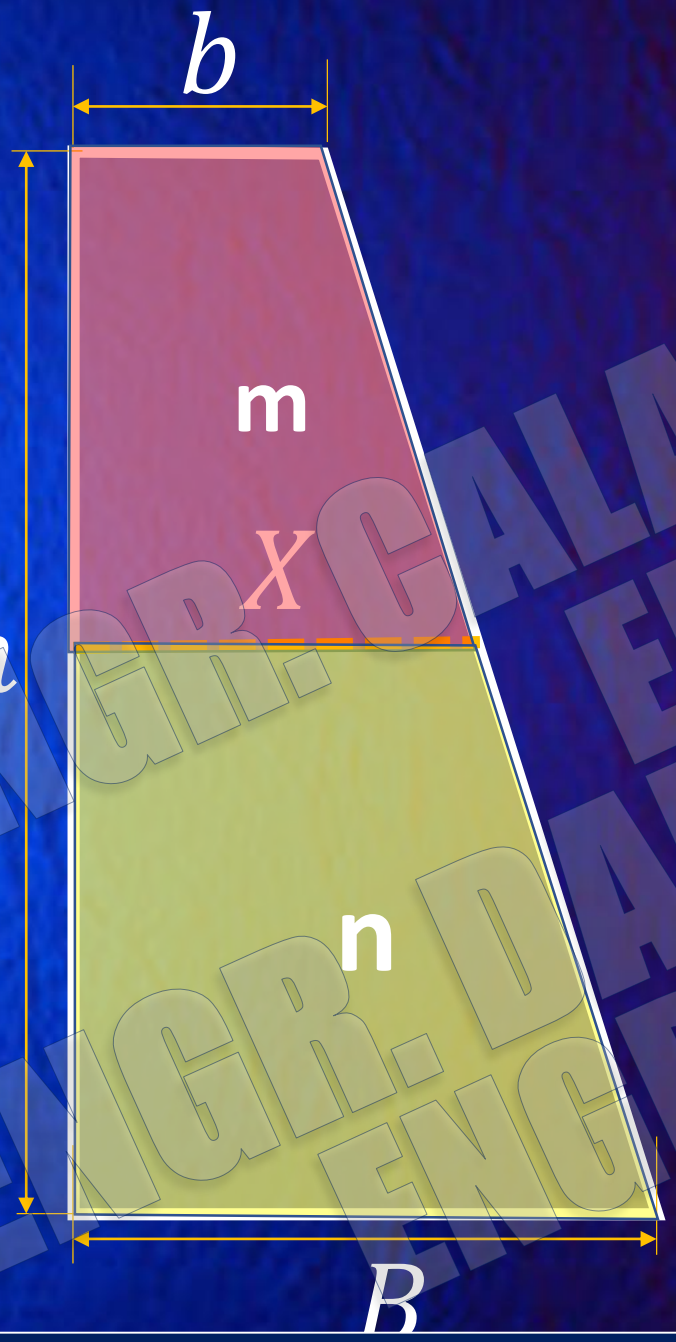
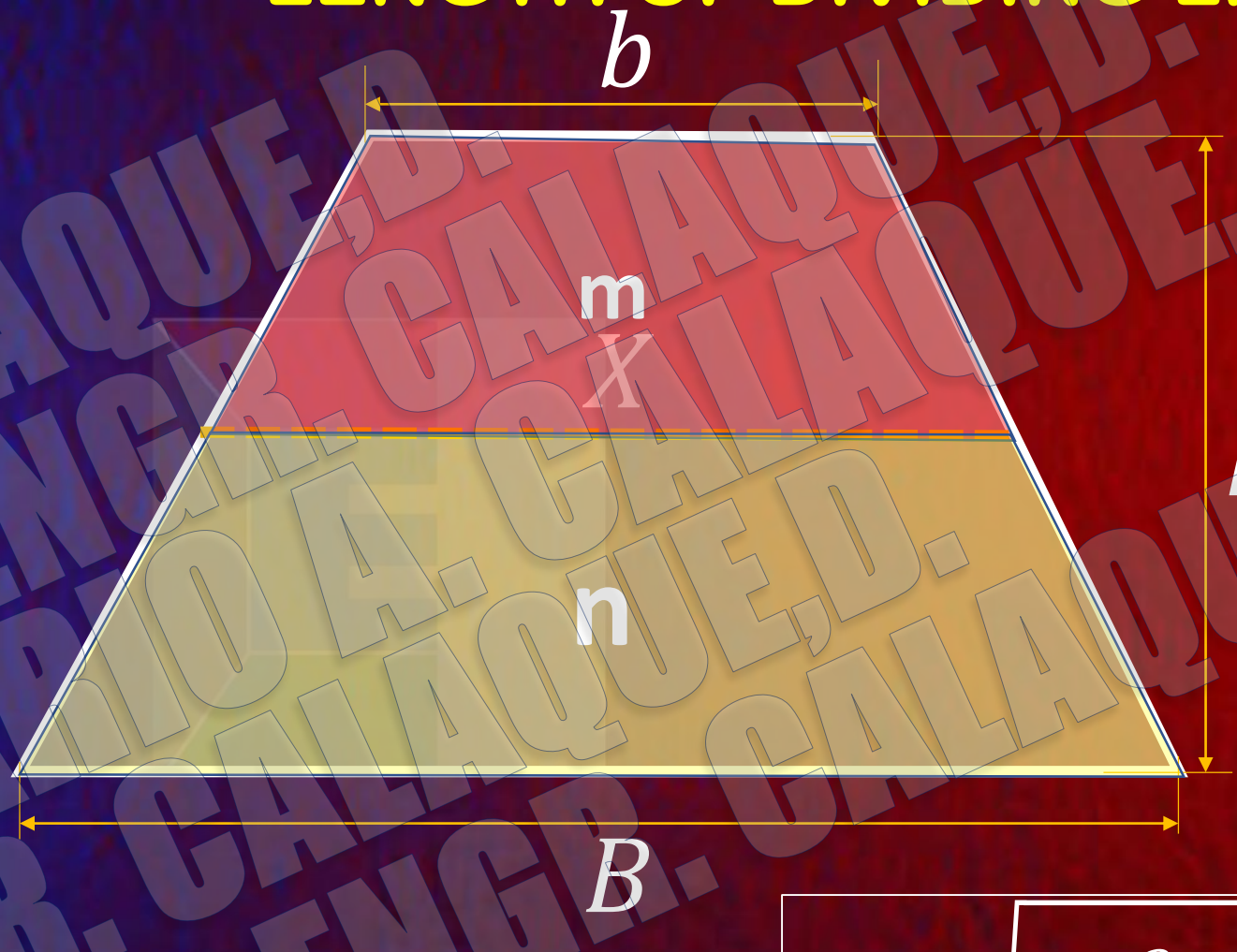
LENGTH OF DIVIDING LINE





$$X = \sqrt{\frac{mB^2 + nb^2}{m+n}}$$

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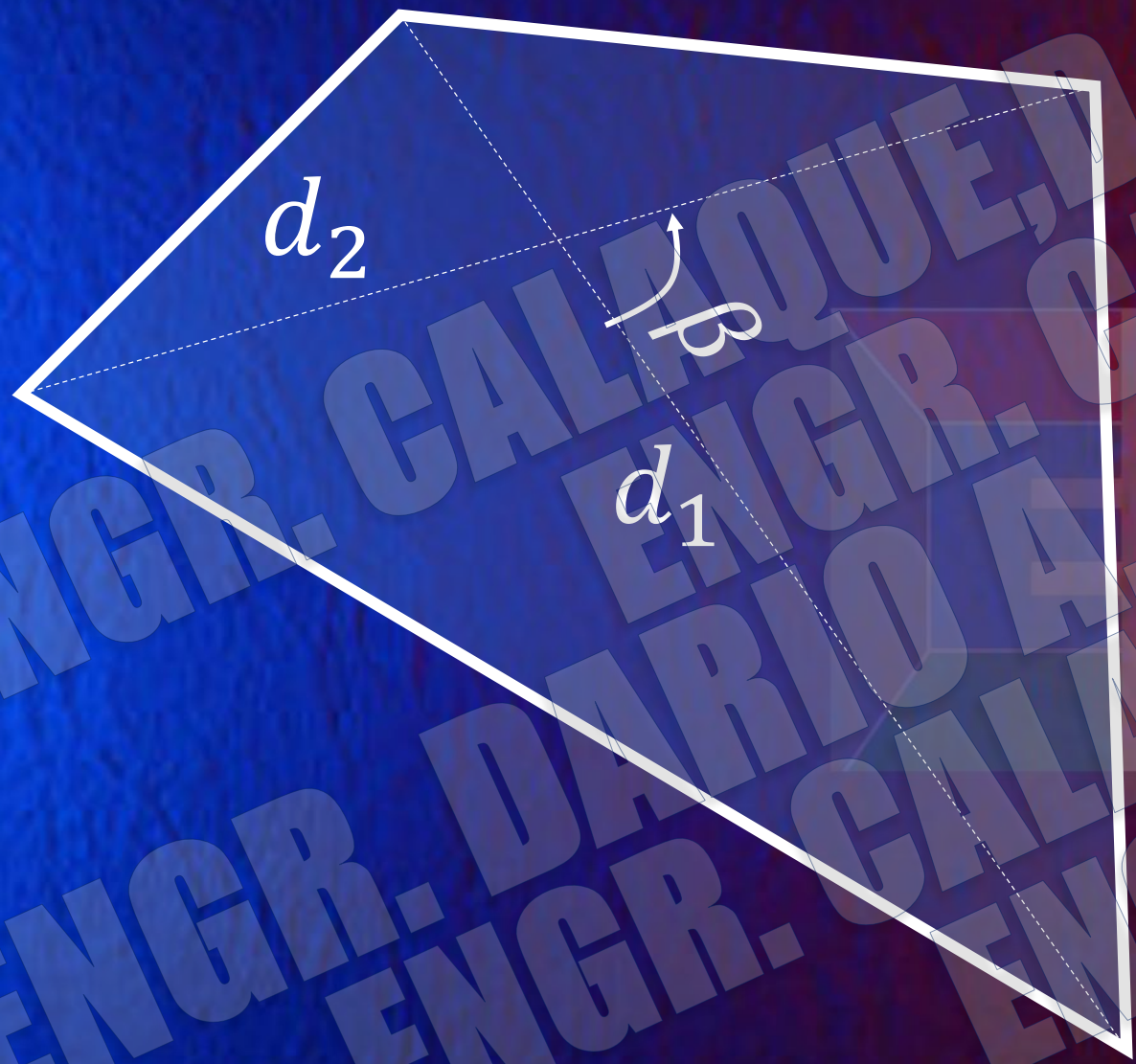
QUADRILATERALS

APPLICABLE TO SQUARE, RECTANGLE, RHOMBUS, PARALLELOGRAM AND TRAPEZOID



$$A = \frac{1}{2}(d_1 d_2)\sin\beta$$

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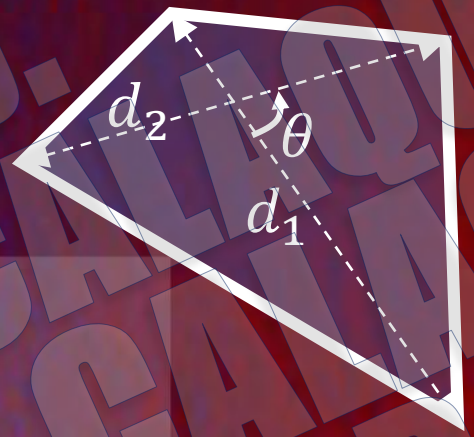
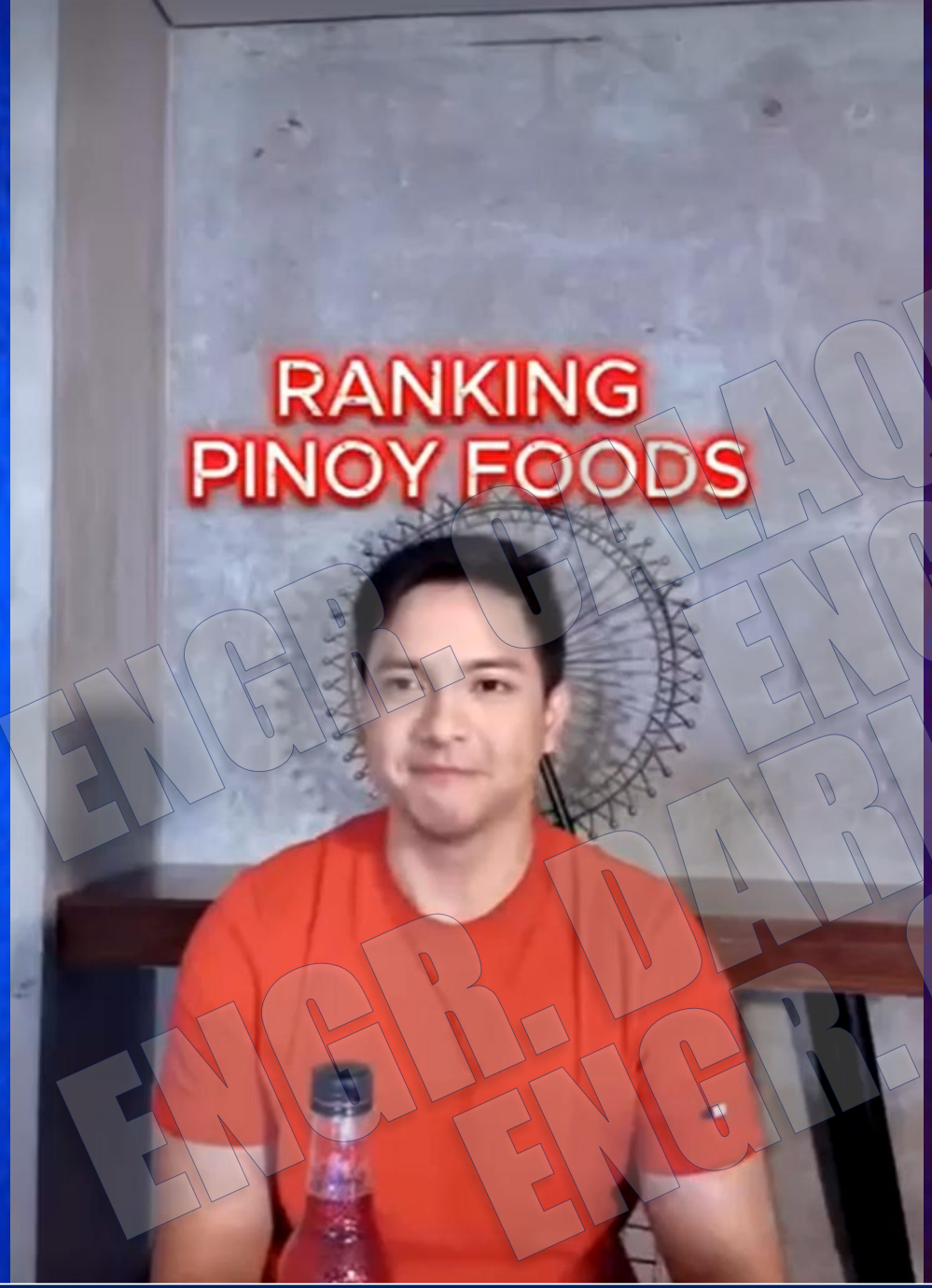
QUADRILATERALS



$$A = \frac{1}{2}(d_1 d_2)\sin\theta$$



Sino tong dila nang dila? Alden Richards

$$\left(\frac{\sin\theta}{2}\right)(d_{\text{ila}}\ d_2^{\text{ila?}}) = A^{\text{lden? Richards}}$$

Sino (over $\sin\theta$) tong (under 2) nang (over d_2)

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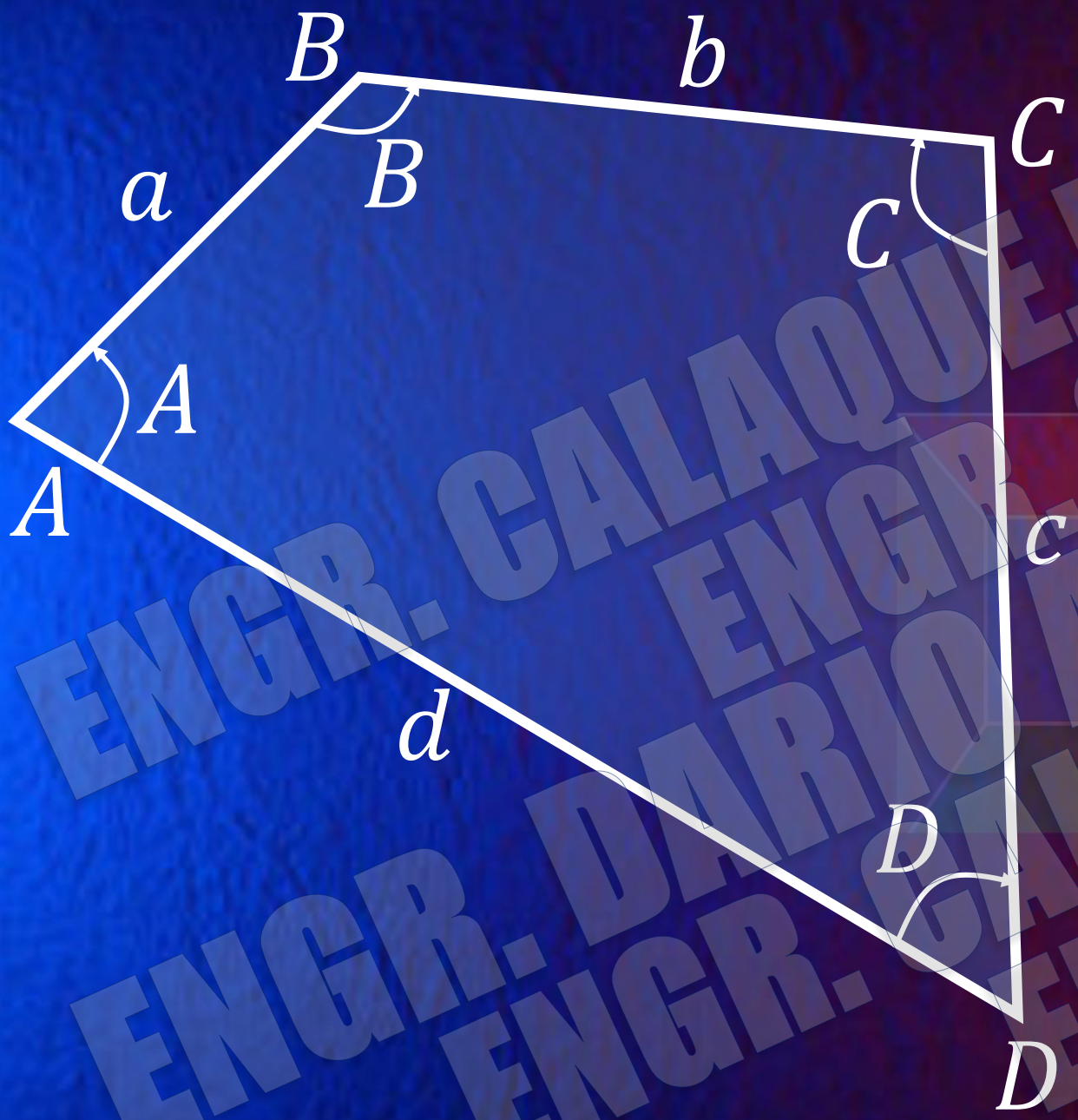

QUADRILATERALS

BRETSCHNEIDER'S FORMULA

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\left(\frac{\emptyset}{2}\right)}$$

$$s = \frac{1}{2}(a+b+c+d)$$

$$\emptyset = A + C = B + D$$

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CELE NOV.2015:

Given triangle ABC, point E is along side AC and point D is along side BC. Line BE and AD are the angle bisectors of angle B and A, respectively. If the angle BEC = 75degrees and angle ADC = 84degrees, find the angle ACB

A) 62DEG

B) 74DEG

C) 44DEG

D) 45DEG

$\alpha = 22$

$\beta = 31$

$\theta = 37$

$\angle ACB = 2\theta = \angle C$

$\triangle BEC$:

① $\beta + 75 + 2\theta = 180$

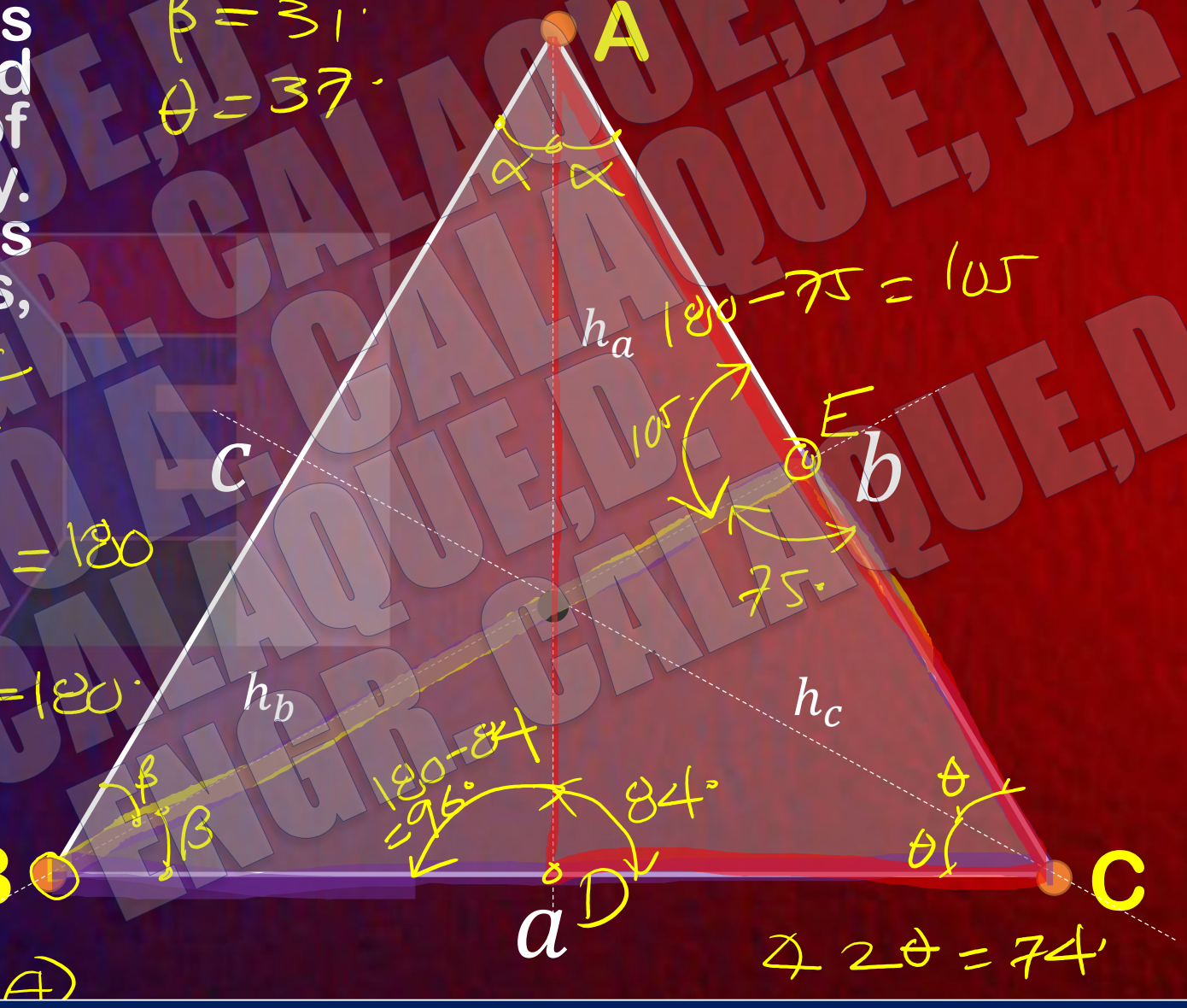
$\triangle ADC$:

② $\alpha + 84 + 2\theta = 180$

$\triangle ADB$:

③ $\alpha + 96 + 2\beta = 180$

$\triangle AEB$: $2\alpha + 105 + \beta = 180$ ④



$\angle 2\theta = 74$

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PROBLEM:

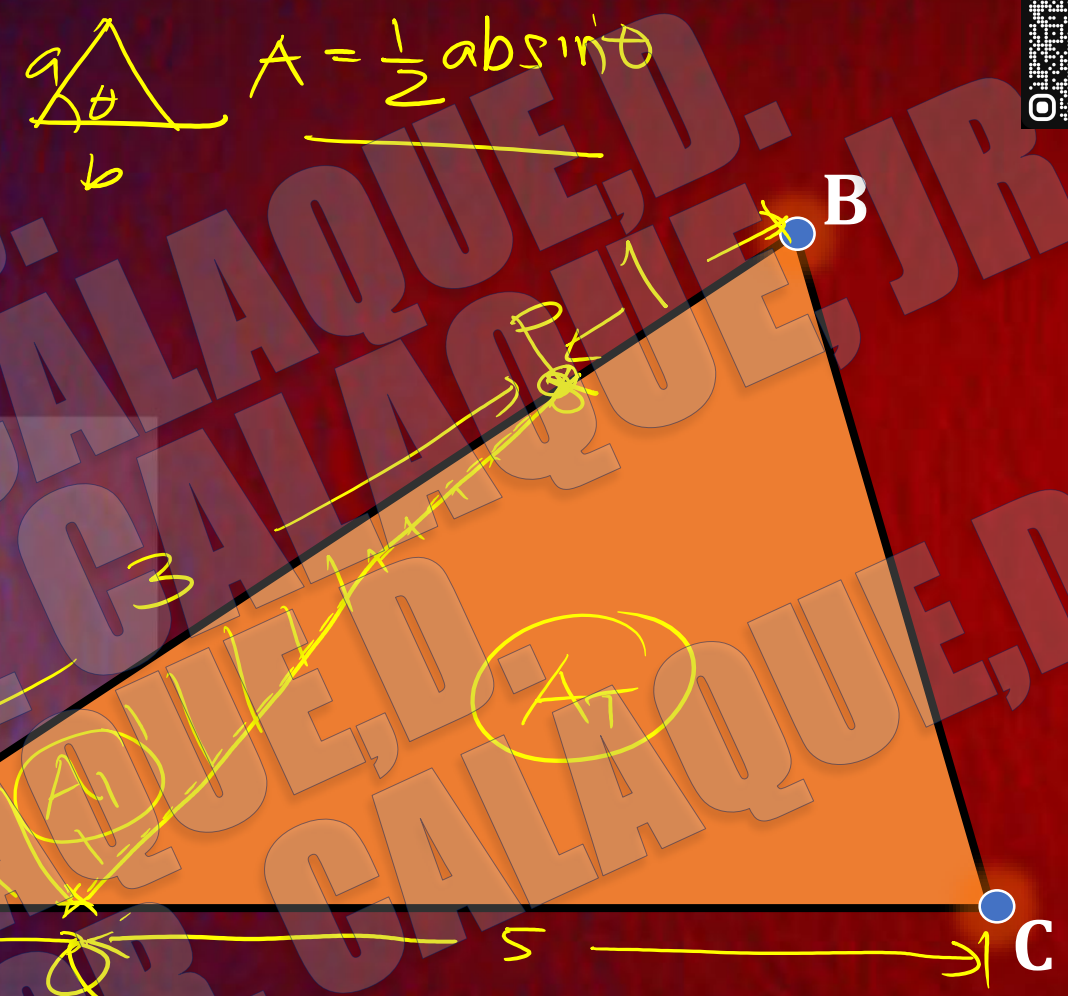
A line is drawn from point P to point Q, where P is along the line AB and Q is along the line AC of the triangular lot ABC. If AP:PB = 3:1 and AQ:QC = 2:5, what is the proportion of the area of APQ and ABC?

A) 1/5

B) 2/7

C) 3/14

D) 5/17



$$\frac{A_1}{A_T} = \frac{\frac{1}{2}(3)(2)\sin\theta}{\frac{1}{2}(4)(7)\sin\theta} = 3/14$$

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PROBLEM:

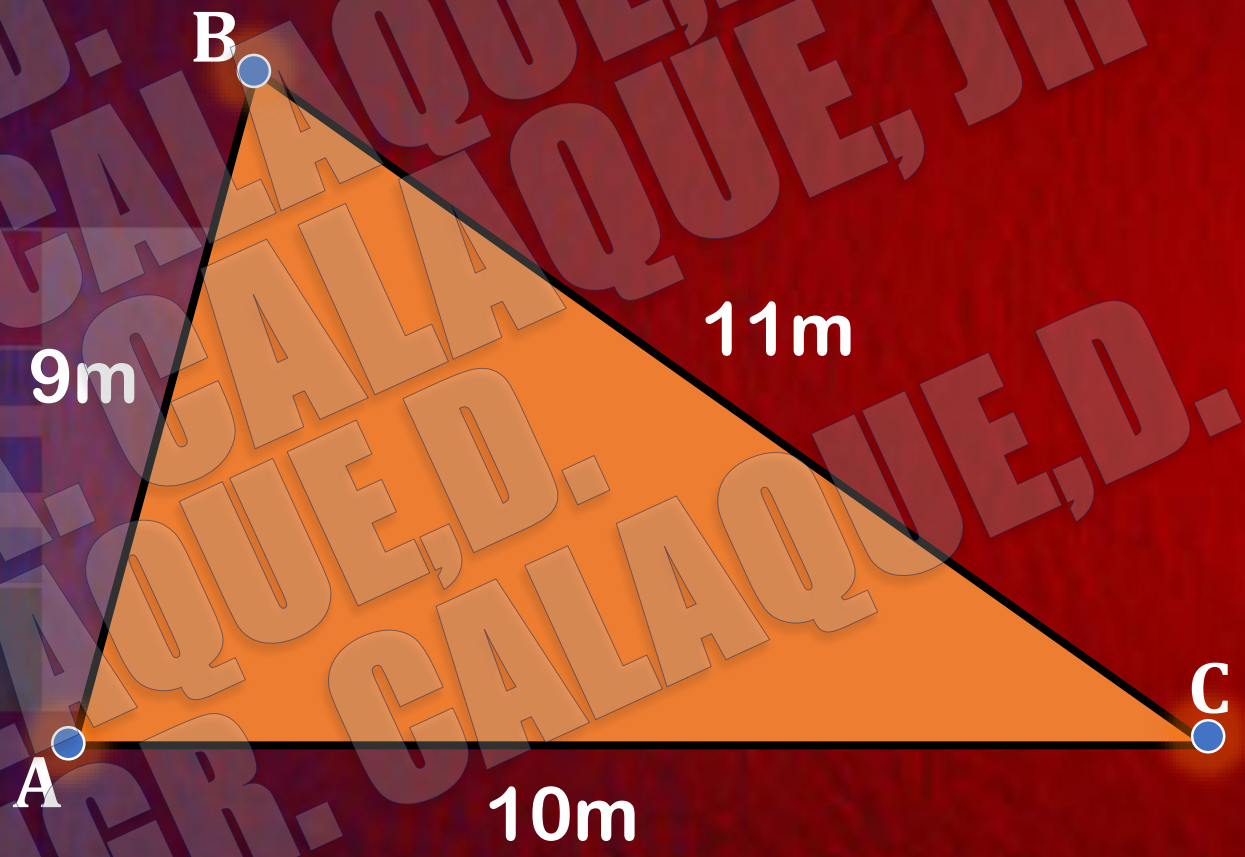
The sides of a triangle are 11, 10 and 9.Find the area of the triangle.

A) 42.4SQ.M

B) 56.7SQ.M

C) 78.3SQ.M

D) 25.5SQ.M



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***Conventional Solution*:**

Area of the triangle

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

$$= \frac{1}{2}(11+10+9)$$

$$= 15m$$

$$A = \sqrt{15(15-11)(15-10)(15-9)}$$

$$\boldsymbol{A = 42.4264m^2}$$

$$(\boldsymbol{answer})$$

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PROBLEM:

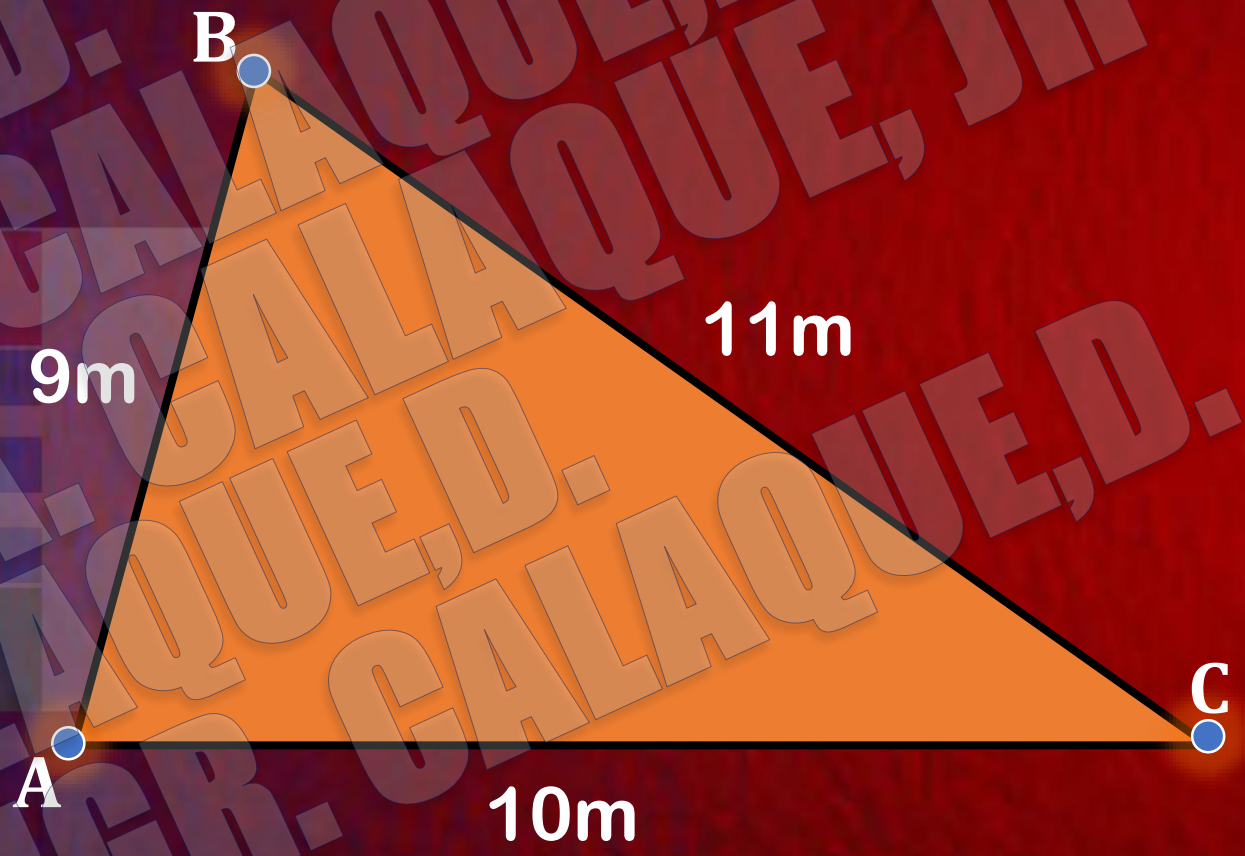
The sides of a triangle are 11, 10 and 9. Find the altitude to the longest side.

A) 6.514M

B) 7.714M

C) 8.914M

D) 10.214M



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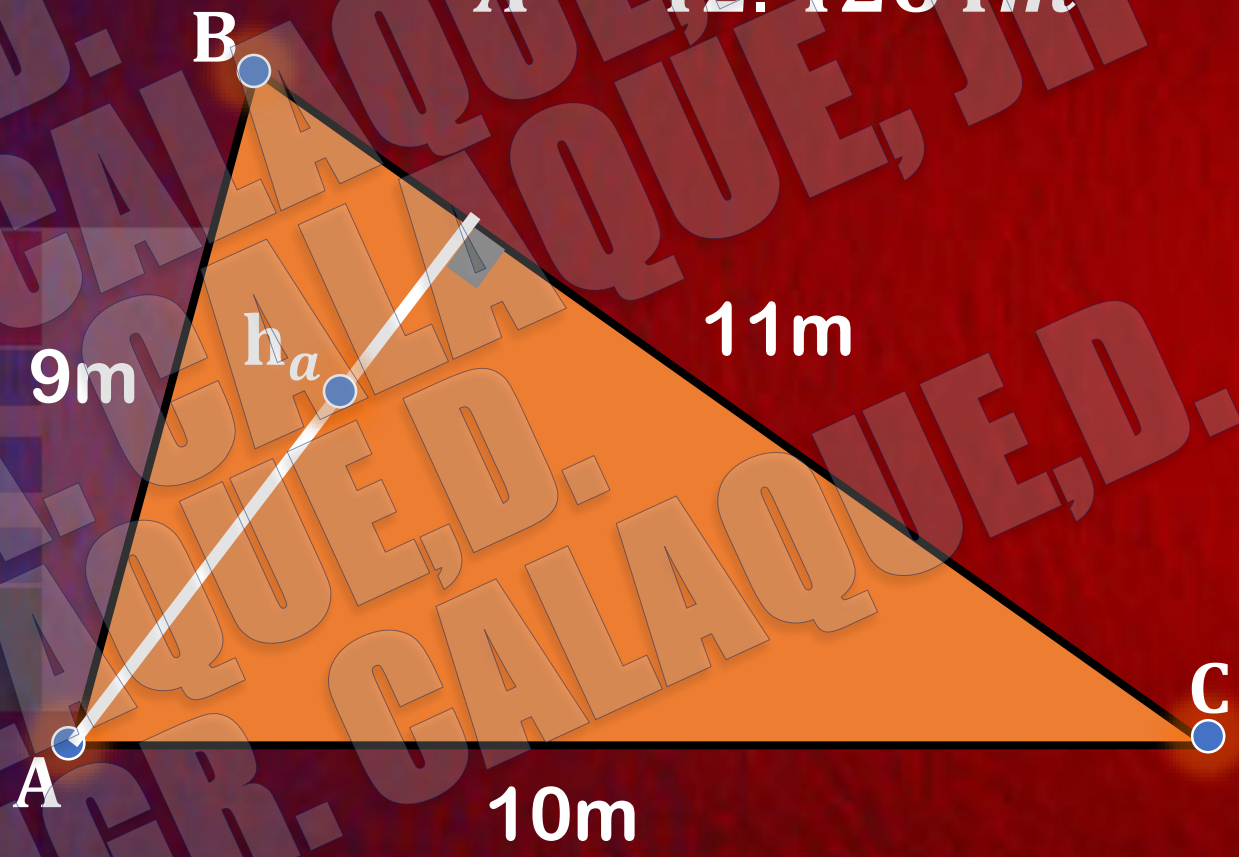
***Conventional Solution*:**
Altitude to the longest side.

$$A = \frac{1}{2}bh$$

$$42.4264 = \frac{1}{2}(11)(h_a)$$

$$\therefore h_a = 7.714m$$
$$(answer)$$

$$A = 42.4264m^2$$



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PROBLEM:

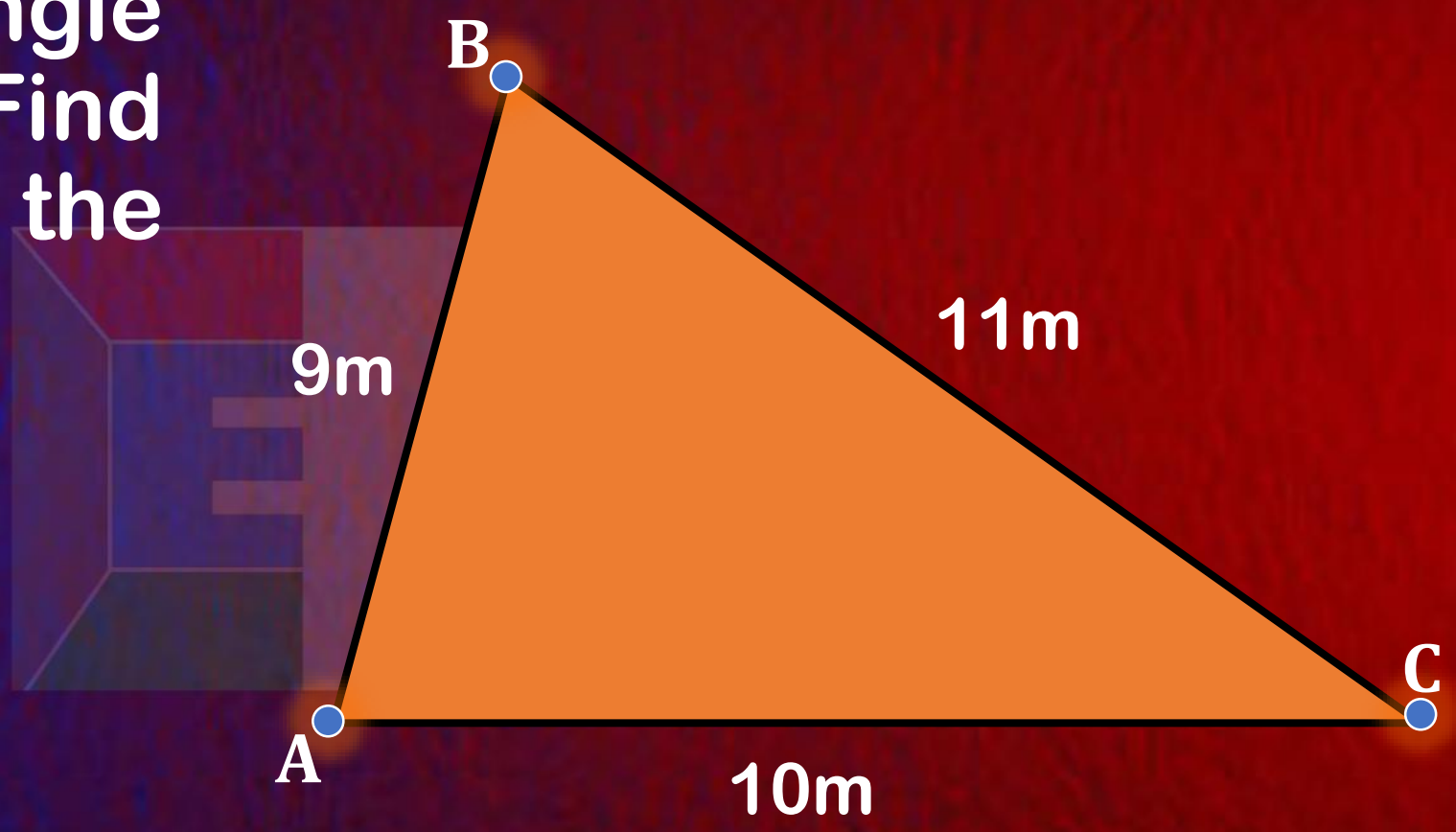
The sides of a triangle are 11, 10 and 9. Find the median to the shortest side.

A) 8.5M

B) 12.5M

C) 11.5M

D) 9.5M



Conventional Solution:
Median to the shortest side.

COSINE LAW:

$$10^2 = 9^2 + 11^2 - 2(9)(11)cosB$$

$$cosB = 17/33$$

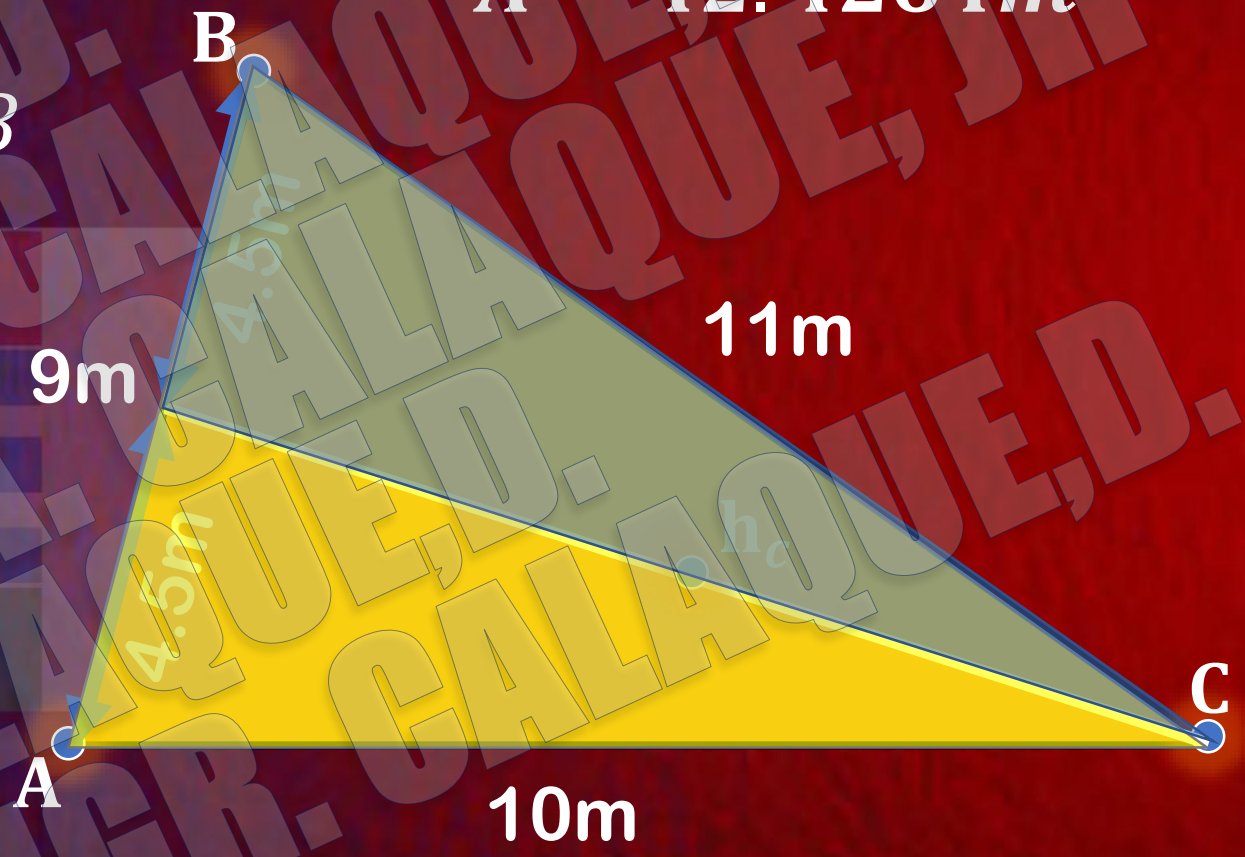
COSINE LAW:

$$(h_c)^2 = 4.5^2 + 11^2 - 2(4.5)(11)\left(\frac{17}{33}\right)$$

$$\boldsymbol{h_c = 9.5\text{m}}$$

$$(answer)$$

$$A = 42.4264m^2$$



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***Conventional Solution*:**

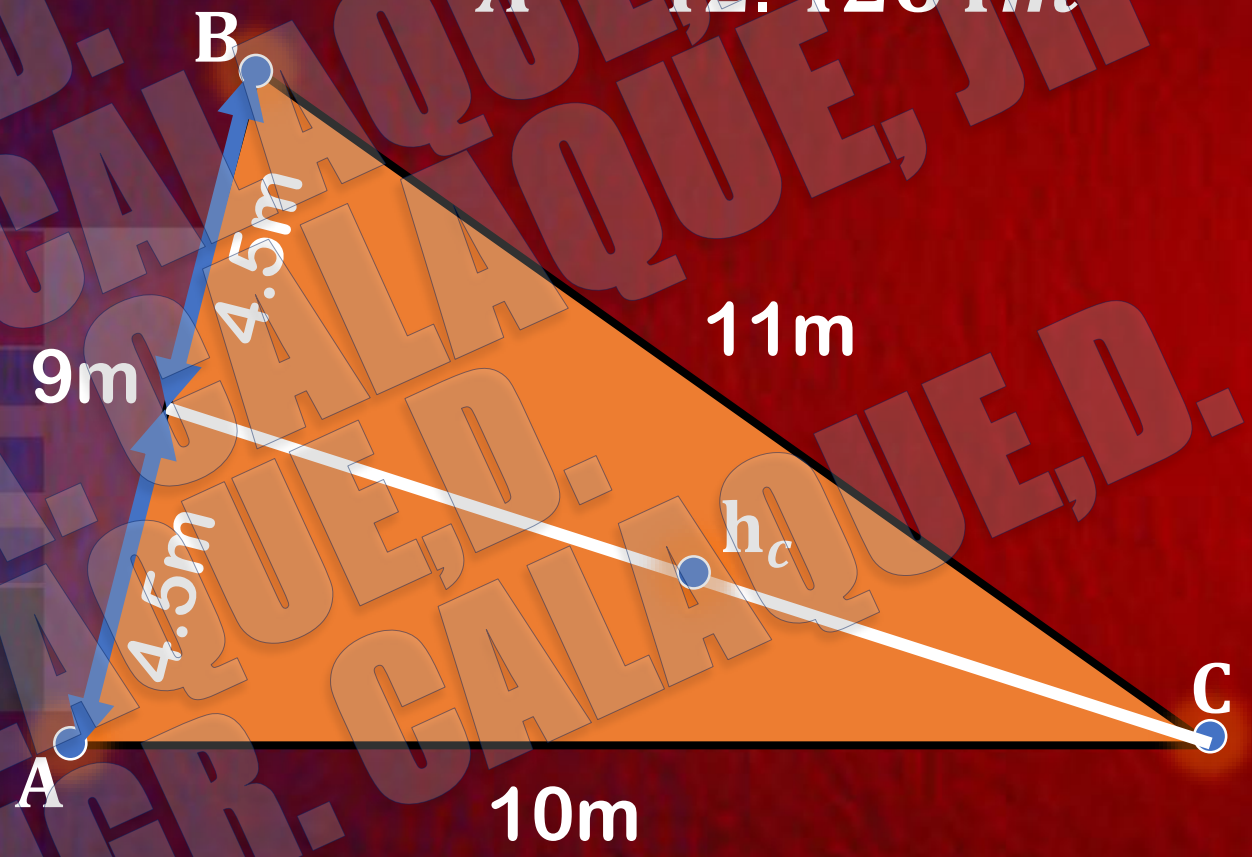
Median to the shortest side.

$$h_c = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$$

$$h_c = \frac{1}{2}\sqrt{2(11^2) + 2(10^2) - 9^2}$$

$$\therefore h_c = 9.5m$$
$$(answer)$$

$$A = 42.4264m^2$$



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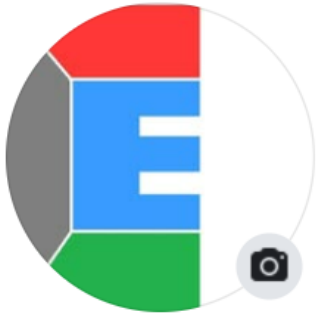

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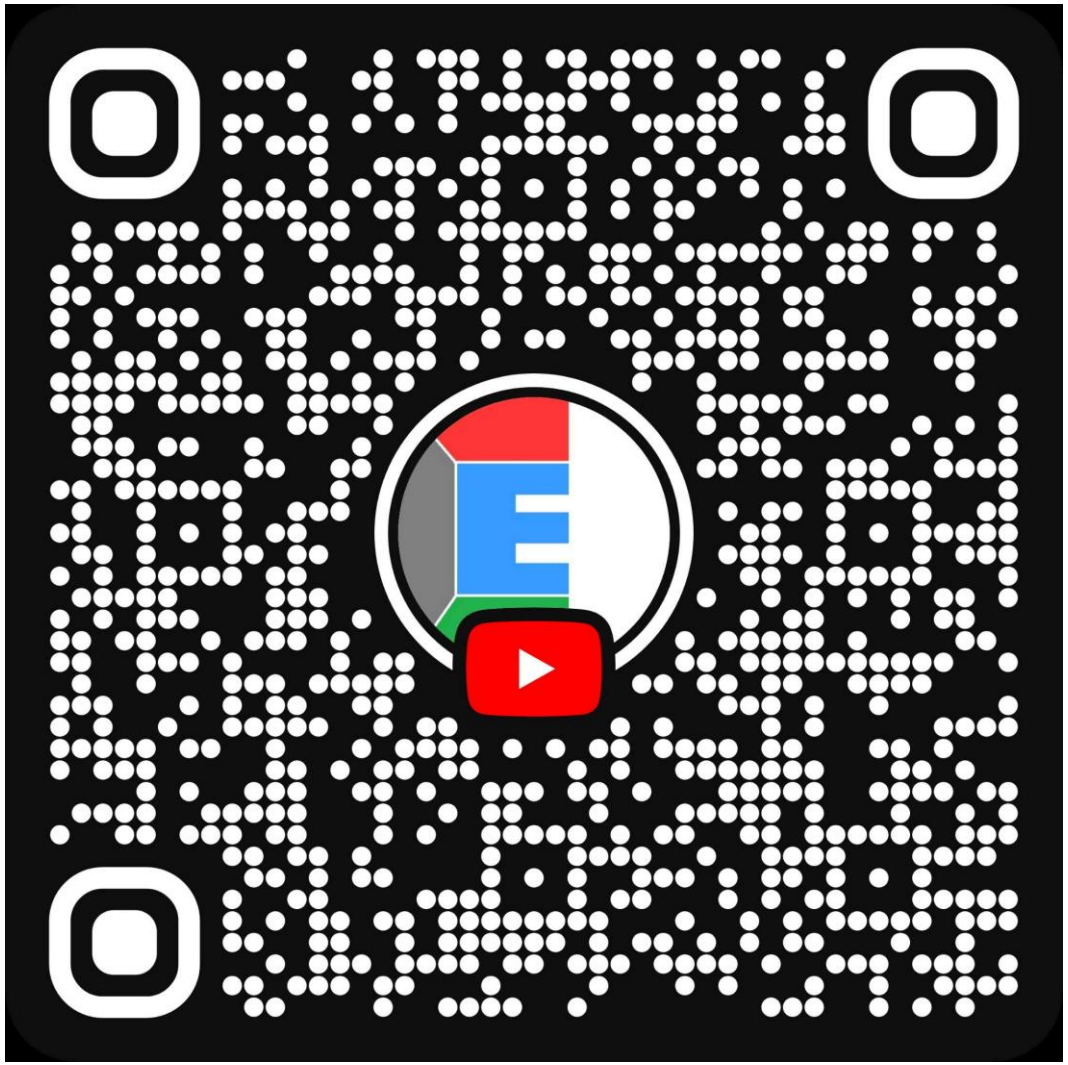

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